

# Beta Is Still Alive!\*

Yexiao Xu  
and  
Yihua Zhao

School of Management  
The University of Texas at Dallas

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## Abstract

This paper investigates whether beta can predict the expected return after controlling for the beta instability resulting from shift in the covariance structure. Such a shift is driven by idiosyncratic volatility's clientele effect: speculative investors prefer stocks with high idiosyncratic volatility. Consequently, these stocks tend to have low future returns from overpricing, and high beta because clientele-based trading also contains systematic component. Indeed, we find that the beta estimate of the current period is positively related to the beta estimate and negatively related to the idiosyncratic volatility measure of the last period. More important, different from existing studies, we find that beta estimates of the current period can significantly explain the cross-sectional differences in future returns of individual stocks, when allowing for an interaction between the current idiosyncratic volatility and the beta estimates. We also show that our simple model can predict the historical expected return well. All results are robust with respect to different measures of beta and idiosyncratic volatility and to different subsamples.

Key Words: Expected Return, Idiosyncratic Volatility, Beta Instability, and Misspricing

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## Abstract

This paper investigates whether beta can predict the expected return after controlling for the beta instability resulting from shift in the covariance structure. Such a shift is driven by idiosyncratic volatility's clientele effect: speculative investors prefer stocks with high idiosyncratic volatility. Consequently, these stocks tend to have low future returns from overpricing, and high beta because clientele-based trading also contains systematic component. Indeed, we find that the beta estimate of the current period is positively related to the beta estimate and negatively related to the idiosyncratic volatility measure of the last period. More important, different from existing studies, we find that beta estimates of the current period can significantly explain the cross-sectional differences in future returns of individual stocks, when allowing for an interaction between the current idiosyncratic volatility and the beta estimates. We also show that our simple model can predict the historical expected return well. All results are robust with respect to different measures of beta and idiosyncratic volatility and to different subsamples.

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# 1 Introduction

The Capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and Black (1972) predicts that differences in the expected returns of individual securities are completely determined by the covariance based beta measure of risk. Many empirical studies including Fama and French (1992), however, provide no or weak evidence to support this prediction. In this study, we show that the lack of empirical evidence is largely due to short-run shifts in the covariance structure between individual stock return and the market return. We further identify that such a shift can be predicted by idiosyncratic volatility. Consequently, we provide strong evidence in supporting a positive relation between the beta measure of risk and the expected return once controlling for the interaction between beta and idiosyncratic volatility.

Although the classical CAPM model is an equilibrium model, it is static in nature with a constant beta measure of risk for each security. There are several reasons, however, to believe that the covariance based beta measure of risk might shift in structure over time. Perhaps, Merton (1973) is the first one to propose a model that allows for time-varying risk as a result of changes in the investment opportunities. Using labor income as a proxy for time-varying investment opportunities, Jagannathan and Wang (1996) provide evidence on the validity of the conditional CAPM.<sup>1</sup> Alternatively, one can treat the position held by equity-holders as a call option on the firm's total asset (Black and Scholes, 1973) since they have the limited liability and the residual claim. Following the idea, Galai and Masulis (1976) (also see Berk, Green, and Naik, 1999) have shown that the equity beta will vary not only with leverage, but also with the volatility of underlying assets even when the asset beta is stable. Bernardo, Chowdhry, and Goyal (2007) and Da, Guo, and Jagannathan (2011) have provide some empirical evidence in supporting this view.

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<sup>1</sup>Lewellen and Nagel (2006) find that the covariance between time-varying beta and time-varying risk premium is too small to explain deviations from the CAPM.

In this paper, we propose and test an alternative explanation for the failure of the beta measure of risk to differentiate the cross-sectional return differences of individual securities. Using daily returns within a month to estimate a monthly beta measure of an individual stock, we find that the current and the next month beta estimates are weakly correlated and vary a lot (with an average autocorrelation being less than 25%). This means that even when the CAPM holds month by month, it is difficult to use the past beta measure to predict next month returns. Such a large instability in the beta measure is unlikely to be a result of changing fundamental risks of a firm which tend to occur over a longer period. Estimation errors are also implausible to account for such large changes in the beta measure since betas are estimated using high frequency returns. One possible explanation is the speculative investment behavior of both institutions and individual investors who tend to chase certain stocks. As a result, not only these stocks tend to be over- or under-priced relative to their rational prices, but the collective activities of these investors will also move the market in the same direction as well. Therefore, the current beta estimates of these stocks will rise temporarily. Moreover, if over-pricing reverses with a lag because of short-sale constraints, returns tend to drop in the subsequent periods. Such a negative relation between current beta and future returns will obscure the true CAPM relation.

There are ample evidence to support our view. Using mutual fund equity holdings data, Falkenstein (1996) shows that mutual funds have a significant preference towards stocks with high visibility and are averse to stocks with low idiosyncratic volatility. Based on the Japanese experience from 1975 to 2003, Chang and Dong (2006) find that institutional herding is positively related to idiosyncratic volatility. For individual investors, Han and Kumar (2008) have shown that retail investors prefer to hold and actively trade high idiosyncratic volatility stocks due to their propensity to speculate. Such special preference by investors' will not only move the prices of these individual stocks but will move the market as well. Using both the

TAQ and ISSM data, Barber, Odean, and Zhu (2009) show that not only individual investors' trading tends to be correlated, but their coordinated trading move the market in a substantial way that causes significant future return difference between heavily bought and heavily sold stocks. One thing in common from all these studies is investors' preference toward stocks with high idiosyncratic volatility. If this is the case, the possible instability of the beta measure should be related to idiosyncratic volatility. Following this logic, idiosyncratic volatility can predict changes in beta estimates, which means we might be able to restore the CAPM relation once controlling for the instability issue.

Indeed, for stocks with large betas and high idiosyncratic volatilities in the current month, future betas tend to be low. At the same time, these stocks also appear to have low future returns due to limited arbitrage (see, Ang, et. al., 1996). In other words, contemporaneously, the CAPM relation seems to hold because small (large) beta seems to be associated with low (high) return. However, the predictive cross-sectional regression will fail. After controlling for the interactive effect between beta and idiosyncratic volatility, we show that the rolling beta measure estimated based on the past monthly returns (see Fama and French, 1992) can explain the cross-sectional return differences of individual stocks. Moreover, if it is these stocks with high current betas and large idiosyncratic volatilities but low future returns that obscure the true beta and return relation, we should expect to see that the CAPM relation holds for the rest of the stocks. After deleting 10% of the stocks with the largest beta and idiosyncratic volatility (accounting for 5% of the market capitalization), the 25 size and book-to-market sorted portfolio returns using the remaining stocks are significantly and positively related to their betas. Our approach not only restore the CAPM relation in a simple way, but also shows the importance of accounting for the instability in beta estimate. To the very least, the CAPM holds in a first order.

Our findings are robust. Both portfolio analysis and Fama-MacBeth regression analysis provides consistent conclusions. While all previously documented firm-level variables, such as the book-to-market ratio, Amihud illiquidity measure, momentum, and return reversal, have significant explanatory power for stock returns, they do not subsume the predictive power of the conventional beta measure for expected return once we control for the interactive effect between beta and idiosyncratic volatility. Moreover, our results are also robust to both the *NYSE/AMEX* market subsample and the *NASDAQ* market subsample, and to the two evenly split subsample periods from 1963 to 1986 and from 1987 to 2010. In all these cases, we continue to find both significant explanatory power of the beta variable and the interaction term between beta and idiosyncratic volatility for the cross-sectional return differences among individual stocks.

In addition, our results are insensitive to different beta estimates. For example, Fama and MacBeth (1973) use a two-step procedure to enhance the power of tests by reducing the noise in the beta estimates. In particular, Fama and French (1992) use the post-sorting portfolio beta estimates instead of the pre-sorting firm-level beta estimates. Recently, Ang, Liu and Schwarz (2010) argue that portfolio beta estimates conceal important information contained in the individual stocks' betas. Therefore, our main results are based on the rolling beta estimates of individual stocks. This choice is also motivated by our argument of instable betas. In the robustness section, we also apply the Lewellen and Negal's (2006) short window beta estimator and the Fama and French's (1992) portfolio beta in the cross-sectional regressions. In all these cases, we consistently show that our main finding of significant effect of beta on the expected return is not altered when using portfolio betas provided that we continue to control for the interaction between beta and idiosyncratic volatility.

This study is also related to several studies on the pricing of idiosyncratic risk. Using the

realized idiosyncratic volatility measured estimated from daily returns, Ang, Hodric, Xing, and Zhang (2006) document a negative relation between current idiosyncratic volatility and the next month return. No matter whether such a negative relation is due to return reversal (Huang, Liu, Rhee, and Zhang, 2010) or gambling (Bali, Cakici, Whitelaw, 2010), it is consistent with our findings. However, we take a step further to examine how idiosyncratic volatility affects the role of beta, and in turn alters future returns. Using alternative measures of idiosyncratic risk, such as the conditional measure (Fu, 2009) or the portfolio measure of idiosyncratic risk (Malkiel and Xu, 2003), others find that idiosyncratic volatility is positively related to future returns, which suggests a pricing effect of idiosyncratic risk. As suggested by Cao and Xu (2009), the priced component of idiosyncratic risk is a relatively small portion due to general diversification effect, mispricing might be a first order effect in short-run. Therefore, we primarily focus on the realized idiosyncratic volatility measure. Other recent studies including Paster and Veronesi (2009) also document the importance of idiosyncratic risk in affecting asset prices. In addition, researchers have found that idiosyncratic volatility is related to the growth option of a firm (see Bernardo, Chowdhry, and Goyal, 2007, Cao, Simin, and Zhao, 2008, Da, Guo, and Jagannathan, 2011, and Johnson, 2004).

We contribute to the asset pricing literature in several important way. First, we show that individual securities' betas vary a lot over time. Such instability makes it difficult for the beta variable alone to predict future returns even if the CAPM holds period-by-period. In Jagannathan and Wang (1996), apart from time-varying betas, the risk premia are also required to change significantly over time in order for the covariance between time-varying beta and time-varying risk premium to be large enough to patch the deviation from the CAPM. In contrast, we only make an effort to predict possible deviations from the CAPM for some stocks directly. Second, motivated by possible investors' preferences toward trading volatile stocks, we find that the realized idiosyncratic volatility is capable of predicting variations

in beta estimates. This means that we are able to predict and adjust deviation from the CAPM for stocks that fail the empirical tests. Finally, as a practical matter, we demonstrate that the simple CAPM model holds for 90% of the individual stocks (or 95% of the market capitalization). At the same time, the beta and return relation holds well for the whole sample of stocks once we control for the interaction between the idiosyncratic volatility and the time-varying beta. In fact, the estimated market risk premium from a multivariate cross-sectional regression resembles the historical average excess return of the market portfolio.

Also related to this paper is the study by Ang and Chen (2007), where they focus on an econometrics approach that explicitly model the dynamics of market risk premium, market volatility, and asset betas. They find that the time-varying beta estimates explains return differences between value and growth stocks. In contrast, we rely on a much simpler approach and be able to show how pervasive is the return and beta relation. The rest of the paper proceeds as follows. In the next section, we describe the data and defines variables used in the study. In addition, we discuss our framework to implement the cross-sectional tests. Section 3 reports our main results. Robustness study is carried out in section 4. Finally, Section 5 concludes.

## 2 Data and Methodology

In this section, we will first motivate our testing strategy. In order to be consistent with existing studies, we also provide information on our sample selection and variable construction in this section.

### 2.1 Methodology

Fama and French's (1992) results are both surprising and controversial. Some researchers argue that both the size and the book-to-market variables are not robust or subject to certain bias.<sup>2</sup> Regardless the merits of these arguments, the beta variable continues to be insignificant in explaining the cross-sectional return differences. Others try to patch the CAPM with different elements. One example is the idea of time-varying risk and risk premium of Merton (1976) as a result of changing investment opportunities. Even when a conditional CAPM model holds perfectly, investors require additional compensation for the covariance risk between time-varying risk and time-varying risk premium. (see Jagannathan and Wang, 1996) Despite the fact that the beta estimate does vary substantially over time, Lewellen and Nagel (2007) find that the covariance between time-varying beta and time-varying risk premium is too small to account for deviations from the CAPM.

Lewellen and Nagel (2007) also find that there is a large variation in the beta estimates. Such large changes in the beta estimates from month to month make it difficult for the beta variable to predict future returns even when the CAPM holds period by period. The large instability in the beta estimates is unlikely to be a result of changes in the fundamental risk (time-varying risk) since it is over relatively short time period. To some degree, Fama and

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<sup>2</sup>An incomplete list includes Ang and Chen (2007), Daniel and Titman (1997), Daniel, Titman and Wei (2001), Horowitz, Loughran and Savin (2000), Knez and Ready (1997), Kim (1997), Kothari, Shanken, Sloan (1995), Loughran (1997), Shumway (1997), Barber and Lyon (1997), Dijk (2011).

Frenh (1992) recognize the issue as a estimation error problem and offer to use portfolio betas as a proxy for individual stocks' betas. We believe that the failure of the beta measure to explain return differences is not mainly an issue of estimation error since we can use high frequency data to estimate beta. Instead, if change in beta has a systematic component and is predictable, we may be able to restore the predictive power of beta as prescribed by the CAPM model. We propose that one possible cause for beta instability is related to investors' speculative trading behavior.

When investors are actively chasing certain stocks, their action will not only affect the prices of these individual stocks but will move the market as well. Consequently, the covariance based beta estimates for these stocks will systematically deviate from their fundamental values. By its nature, such a deviation will not alter the long-term expected return of a firm and is likely to revert in near future. In other words, individual stocks' return may still be contemporaneously correlated with the market return, which makes the market factor remain to be the single most powerful factor in explaining time-series asset returns, while the beta measure is incapable to differentiate the cross-sectional return differences.<sup>3</sup> In addition, the nature of these deviations suggests that they are not necessarily covary with macro factors, which makes it difficult to correct by relying on the idea of time-varying betas. This may be the reason that, even considering the time-varying factor, the conventional measure of beta is insignificant in cross-sectional tests. Note for a particular stock, such a deviation may be temporally. For the market as a whole, there exist such deviations in the beta estimates at any given point of time, but for different stocks. Therefore, instability is likely to be pervasive.

There are ample evidence supporting that speculators tend to focus on stocks with large idiosyncratic volatilities. For example, using retail level data, Han and Kumar (2008) have

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<sup>3</sup>In fact, these stocks tend to have low future returns because of temporally increase in prices despite increases in betas, which further weakens the possibility of finding a positive relation between beta and future return.

shown that investors prefer to hold and actively trade high idiosyncratic volatility stocks due to their propensity to speculate. On the institutional investor level, Falkenstein (1996) shows that mutual funds have a significant preference towards stocks with high visibility and are averse to stocks with low idiosyncratic volatility. Using Japanese data from 1975 to 2003, Chang and Dong (2006) find that institutional herding is positively related to idiosyncratic volatility. No matter who is trading, the heavy trading activity will not only affect the prices of these stocks but move the overall market as well since individual investors trading on these stocks tend to be correlated. Indeed, using both the TAQ and ISSM data, Barber, Odean, and Zhu (2006) document that not only individual investors' trading tends to be correlated, but their coordinated trading move the market in a substantial way that causes significant future return difference between heavily bought and heavily sold stocks. Consequently, the increased comovement of these stocks with the market will induce temporary increases in the beta estimates of these stocks. If the future expected return of an individual stock is still determined by its true beta, we will find a weaker relation between the current beta estimate and future returns. Therefore, stocks with large idiosyncratic volatilities are more likely to experience large deviation in their beta estimates. At the same time, as Ang, Hodrick, Xing and Zhang (2006) have shown that stocks with large idiosyncratic risks are more likely to have low future returns, other things being equal.<sup>4</sup> These two factors—deviation in betas and low future returns will obscure the true beta and return relation as predicted by the CAPM.

Without the deviations in beta and the low return of some volatile stocks, the CAPM relation may hold well, at least in the first order. A simple control for low future returns using idiosyncratic volatility in the cross-sectional regression won't solve the problem. As our analysis above suggests that stocks with large idiosyncratic volatilities are more likely to have

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<sup>4</sup>It is also reasonable to argue that overpricing is more likely to prevail than underpricing due to high arbitrage costs created by large idiosyncratic volatility (see Shleifer and Vishny, 1997). Therefore, stocks with large idiosyncratic volatilities may have low future returns.

positive deviation in their beta estimates, we propose to control for the beta instability using an interaction term between beta and idiosyncratic volatility in the cross-sectional regression. Since these stocks with both large beta and idiosyncratic volatility tend to have low future returns, we hypothesize that the interaction term in the regression will not only have a negative sign but also make the beta variable itself positive and significant. Idiosyncratic volatility is a key variable in solving the beta-return puzzle of the CAPM, we will further investigate its ability to predict variations in beta estimates.

## 2.2 Data Sample

Similar to most studies in asset pricing, our sample covers stocks traded on the NYSE, AMEX, and NASDAQ exchanges over the sample period from July 1963 to December 2010. This choice of the sample period also reflects the availability of daily returns. All stock returns are obtained from the Center of Research in Security Price (*CRSP*), while factors returns are obtained from Kenneth French's website. As a common practice, our sample of stocks is restricted to ordinary common stocks with share code 10 and 11. Financial firms, ADRs, shares of beneficial interest, companies incorporated outside U.S., American Trust components, close-ended funds, preferred stocks, and real estate investment trusts (REITs) are excluded from our sample. Accounting measures, including book-to-market, are constructed according to Fama and French (1992) using the *COMPUSTAT* database. To ensure that we have all the information for each stock, we use the merged *CRSP* and *COMPUSTAT* database.

In cross-sectional regressions, at any given month, we require all firms in our sample to have data in all the variables. As a result, we have over 5000 firms each month on average. In order to avoid possible outliers or influential observations, we apply the following filters to the population of firms. In particular, we winsorize all the variables each month at the 0.5%

and 99.5% level to control for the potential data errors the effect of extreme values on the coefficient estimates. To ensure the robustness of our results, we also split the whole universe of stocks into the *NYSE/AMEX* subsample and the *NASDAQ* subsample, and divide the whole sample period into two equal subsample periods of 1963-1986 and 1987-2010.

### 2.3 Variables

In order to be comparable with the original study by Fama and French (1992), we follow their approach in estimating the key variable, beta. In particular, it is estimated from a market model based on the past 24- to 60-month returns (as available) to accommodate the feature of time-varying beta. It is denoted as  $Beta_r$ . At the same time, such a measure might also capture the instability in the beta estimates. Fama and French also use the post-ranking beta estimates ( $Beta_p$ ) to alleviate possible large estimation error. Such a noise in an individual stock's beta estimate will bias down the regression coefficient in the second-stage cross-sectional regression due to the error-in-variables problem. We thus also estimate the 100 portfolio betas, where portfolios are constructed by sorting individual stocks based on their size and pre-ranking betas at the beginning of June each year. Since the post-ranking betas of individual stocks are obtained by reassigning the portfolio beta to individual stocks within the portfolio, the post-ranking beta may still reflect beta instability to some degree.<sup>5</sup> These two methods of estimating beta may not be powerful enough to capture all the features of beta instability since both measures rely on the long window (more than 5 years return data) to estimate. Lewellen and Nagel(2006) has proposed an alternative approach by using short-window regressions to estimate beta. They argue that beta estimates from these short horizon regressions are unbiased estimated of the conditional betas. Therefore, we also adopt a similar procedure using the daily returns over the last three months to estimate beta in our

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<sup>5</sup>In fact, whether a stock is belong to a high or low beta group is determined by its pre-ranking beta.

study ( $Beta_{3d}$ ).

Our second key variable is the idiosyncratic risk measure. Following Campbell, Lettau, Malkiel, and Xu (2001) and Ang et al (2006), we use realized idiosyncratic volatility calculated based on the daily residual returns of individual stocks in the last month. In particular, we regress daily excess returns of each stock on the Fama-French three factors plus the Carhart's momentum factor each month as in the following model,

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + s_i r_{SMB,t} + h_i r_{HML,t} + u_i r_{UMD,t} + \epsilon_{i,t}. \quad (1)$$

where  $r_{m,t}$ ,  $r_{smb,t}$ ,  $r_{hml,t}$ ,  $r_{umd,t}$  are returns from the market portfolio, the size portfolio, the book-to-market portfolio, and the momentum portfolio, respectively. The residual square is then summed to compute the idiosyncratic volatility in that month ( $IV_d$ ). On average, there are 21 daily returns each month. In order to have a comparable sample as that used to compute  $Beta_r$ , we also require that there are at least three-month of trading data in order to include the firm in the cross-sectional regression at a particular month.<sup>6</sup> Of course, any estimates of idiosyncratic risk depend on a particular asset pricing model. As a robust check, we also use the total volatilities of individual stocks ( $TV_d$ ) as a proxy since over 80% of the total volatility is idiosyncratic.

We focus on the realized idiosyncratic volatility for two reasons. First, as argued by Merton, the estimate of volatility is more accurate when high frequency return data are used. Second, our use of idiosyncratic volatility is primarily to predict beta instability rather than assessing its pricing effect. If the conditional idiosyncratic volatility measure of Fu (2009) is used, we will not only limit our sample size because of convergence issue but also our ability to capture beta instability. However, we do apply the above model to monthly returns on a rolling basis using the last 24- to 60-months (as available) (see Malkiel and Xu, 2003) to

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<sup>6</sup>In order to reduce the impact of extreme returns and for robustness, we also estimate the idiosyncratic volatility using daily returns in the last three months. Results are generally a little stronger.

obtain the rolling idiosyncratic volatility estimate ( $IV_7$ ) as an alternative measure.

As popularized in the current literature, we construct several control variables that are related to firm characteristics that might be related to the cross-section of expected stock returns. Following the Fama French's (1992), we obtain the market capitalization ( $ME$ ) and the book-to-market ration ( $B/M$ ) of a firm. We also include the one-month lag return of  $Ret(-1)$  to capture the return reversal effect, the lag two-month to seven-month compounded return of  $Ret(-2, -7)$  to control for the momentum effect, and the Amihud (2002) illiquidity measure ( $Illiq$ ) to control for the indirect trading cost. Specifically, the Amihud illiquidity is defined as the average ratio of the daily absolute return to the dollar trading volume in the last month.

### 3 Empirical Results

We provide evidence in supporting the pricing role of beta risk under the framework of beta instability from several perspectives. In order to understand the consequence of beta instability and how it is related to idiosyncratic volatility, we first offer two-way sorting results sorted on beta and idiosyncratic volatility. In cross-sectional analysis, we explicitly control for the interaction between beta and idiosyncratic volatility to show the pricing power of the beta risk. To further demonstrate how instability of beta affects the cross-sectional results, we study the “dynamics” of beta over time. Before getting to the details of our empirical results, we take a brief look at the characteristics of our sample to ensure the comparability of our analysis with the existing studies in the literature.

#### 3.1 Summary Statistics

The summary statistics for variables used in our study is reported in Table 1. Over the sample period from 1963 to 2010, the average monthly return ( $Ret$ ) is 1.2%. Although, it is a little high than the historical norm, this is not the average market return (rather the firm-month average).<sup>7</sup> The median return of 0.0% indicate a skewed return distribution. The average portfolio beta ( $Beta_p$ ) of 1.36 seems to be high, but it is close to that reported in Fama and French (1992). In contrast, the average rolling beta ( $Beta_r$ ) of 1.16 is reasonable (with a median of 1.09). Since the rolling beta is measured on individual stocks, it has more than twice of the variation (0.74) as that of the portfolio beta. As expected, the average beta computed from daily returns ( $Beta_{d,-1}$ ) is much lower (0.737), which is consistent with that reported in Ang, Hodrick, Xing and Zhang (2006). However, the standard deviation is as high

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<sup>7</sup>One can consider the standard deviation of the firm-month observations of 15.9% as the total volatility of return, which is consistent with the finding of Campbell et al (2001) that idiosyncratic volatility accounts for the majority part of the total volatility.

as 1.37. To reduce possible noise, we use  $Beta_{d,-3}$  estimated using past three-month returns in the robust analysis section.

Insert Table 1 Approximately here

Although the average idiosyncratic volatilities calculated using daily returns,  $IV_d$ , and calculated using the past 24 to 60 monthly returns,  $IV_r$ , are very similar (12.7% versus 12.6% per month), the  $IV_d$  measure fluctuate 40% more than the  $IV_r$  measure. Therefore, the realized idiosyncratic volatility measure ( $IV_d$ ) might be easier than the rolling idiosyncratic volatility measure ( $IV_r$ ) to capture the beta instability if there is any. To be expected, idiosyncratic volatility accounts for majority part of the total volatility, which is 15.1% on average. In addition, the average firm size is \$100 million with 25% of the firms have an average market value less than \$20 million, and 25% of the firms have an average market value more than \$440 million. The mean and median of log book-to-market ratios are  $-0.47$  and  $-0.38$ , respectively, indicating a negative skewed distribution in the variable. The mean and standard deviation of the last 2- to 7-month compounded returns are 7.8% and 43.1%, respectively, which are consistent with those of the average monthly return. Also consistent with other studies, the Amihud (2002) illiquidity measure tends to skew to the right. Following the practice in the literature, we control for size, book-to-market, momentum, return reversal, and illiquidity in our cross-sectional regression. The statistics for all these variables are comparable to those reported in the literature.

### 3.2 Portfolio Analysis

Although the market factor seems to be the most important factor in explaining the time-series variation in individual stock returns, beta has been consistently shown to lack the cross-sectional explanatory power for return differences. As discussed in the first section, one

contributing factor might be the instability of beta, which could alter the relation between return and beta. Moreover, as hypothesized that such instability should be related to idiosyncratic volatility. The simplest way to examine a potential complicated relation among beta, idiosyncratic volatility, and expected return is the two-way sorting approach. In order to be consistent with our cross-sectional regressions in the next section, we first sort all stocks into five groups based on their rolling beta measure ( $Beta_r$ ) at the beginning of each month. These groups of stocks are then divided into five sub-quintiles based on their realized idiosyncratic volatility measure ( $IV_d$ ). As a result, we obtain 25 portfolios each month. The average returns of each portfolio are report in Panel A of Table 2.

Insert Table 2 Approximately here

At a first glance, we see that portfolios with large idiosyncratic volatility and large beta have far lower returns than those of small beta and low idiosyncratic volatility portfolios, which is inconsistent with the CAPM prediction. Moreover, portfolio returns seem to vary with idiosyncratic volatility in a systematic way. As shown by Ang, et. al. (2006), the low idiosyncratic volatility portfolio tends to have a high average return than that of the high idiosyncratic volatility portfolio as shown in the last row of Panel A of Table 2. Although the difference between the two extreme portfolio returns is negative, such a pattern is only true when beta is relatively large. In fact, the relation between return and idiosyncratic volatility is hump shape for any given level of beta. This is consistent with Bail, et. al.'s (2007) finding of a non-monotonic relation between idiosyncratic volatility and portfolio returns.

For the beta variable, when idiosyncratic volatility is relative low, portfolio returns increase with beta monotonically as predicted by the CAPM. In fact, the difference between the low and high beta portfolio returns is 0.43% per month and is significant for the low idiosyncratic

volatility group. When idiosyncratic volatility increases, such a monotonic relation starts to reverse. In particular the relation between return and beta looks hump shape for portfolio with median level of idiosyncratic volatility. At the largest idiosyncratic volatility level, we observe a decreasing relation between portfolio return and beta. The difference between low beta and high beta portfolio returns is  $-0.64\%$  and is statistically significant. Therefore, such a complicated nonlinear relation is unlikely to be captured by a single beta or a simple control for idiosyncratic volatility.

In Panel B, we group stocks into portfolios according to their post-ranking betas each month as in Fama and French (1992). The nonlinear relation observed in Panel A for the rolling-beta sorted portfolios repeats to a large extent. Perhaps, the positive relation between portfolio return and beta is even stronger than what is seen in Panel A when idiosyncratic volatilities is relatively low. In addition, the negative relation between portfolio returns and idiosyncratic volatilities is also even stronger when beta is large. Because of the persistent non-monotonic relation between beta and return seen in both panels with different level of idiosyncratic volatilities, we need to control for the interactive effect between beta and idiosyncratic volatility. By doing so, we may restore the first order line relation between beta and portfolio returns.

### **3.3 Fama-MacBeth Regression Results**

Portfolio analysis in the previous section has revealed the nonlinear relation among return, beta, and idiosyncratic risk, which is important in understanding why the beta measure of systematic risk cannot predict expected returns suggested by the existing literature. However, the portfolio sorting approach is often limited by the number of dimensions by which we can sort, and is unable to reveal a true relation when other factors are likely to be important

simultaneously. In order to better isolate the effect of beta on future stock returns and to incorporate the possible time-varying structure, we employ the standard Fama-MacBeth regression approach. In addition, we also control for other known factors that affect the cross-sectional return difference, such as, size, book-to-market, momentum, liquidity, and return reversal. To increase the power of our tests, we focus on individual stocks in our analysis.

Each month, we regress the monthly individual stock returns on the past rolling beta estimate and the past idiosyncratic volatility measure. We further add the interaction term between beta and past idiosyncratic volatility in our regression model. This interaction term captures the possible future beta change due to the instability of the beta measure. The time-series average of coefficient estimates are reported in Table 3 along with the Newey-West (1987) robust  $t$ -statistics to account for the correlation among the estimates.

Insert Table 3 Approximately Here

The results shown under Model 2 and Model 3 in Table 3 confirm the finding in Fama and French (1992) that the beta measure of the systematic risk has no explanatory power for expected return, while variables related to firm characteristics, such as book-to-market, do offer explanatory power. Different from Fama and French (1992), the size variable is insignificant. This is consistent with many other studies that finding weak explanatory power for the size variable in recent years. When including the realized idiosyncratic volatility in Model 4, the size variable becomes very significant again. This might be due to the high correlation between idiosyncratic volatility and the size variable (see Malkiel and Xu, 1997) which helps to reduce the noise in the size variable if size is a proxy for some risk factors. Consistent with Ang, et. al.'s (2006) finding, the realized idiosyncratic volatility is negatively related to future returns. However, the beta measure remains insignificant.

Our portfolio analysis suggests that stocks with both large beta and idiosyncratic volatility tend to have low return. As suggested in section 1, this is largely due to the instability of the beta measure and the likelihood of mispricing for these stocks. In order to test this hypotheses by controlling for such nonlinearity, we include the interaction terms between beta and idiosyncratic volatility in our regression model. Consistent with the CAPM theory, the beta variable becomes very significant with a positive coefficient estimate of 0.5% per month as shown in Model 1. Although this is a little high than the average excess market return of 0.44%, it is a big step forward comparing with that of Fama and French's (1992) estimate. At the same time, the interaction between beta and the realized idiosyncratic volatility is negative and statistically significant. This result further reveals two important features. First, the failure of Fama and French in finding a positive relation between return and beta is not a simple control of missing factors. As shown by the interaction term, the instability in future betas is largely a function idiosyncratic volatility as hypothesized. Second, even when the CAPM holds at any moments, temporary systematic deviation can be large enough to temper the true relation because the systematic risk only counts for a small portion of the total risk of individual stocks.

The cross-sectional regression coefficient for the beta variable remains very significant even after we control for the Fama and French's (1992) size and book-to-market factors, despite the estimate increases to 0.6% as shown in Model 5. Comparing with Model 4, the negative relation between the realized idiosyncratic volatility and future returns is subsumed by the interaction term. In other words, the negative relation is limited to stocks with large beta due to investors' short-term behavior of chasing stocks with high idiosyncratic volatility. In fact, idiosyncratic volatility is positive although insignificant in Model 1. This indicates that idiosyncratic risk might have a pricing effect, but the realized idiosyncratic volatility measure is too noisy to capture the priced component of idiosyncratic risk as argued by Cao and Xu

(2009).

The coefficient estimates drops to 0.4% when controlling for return reversal and momentum as in Model 6, and controlling for liquidity in addition as in Model 7. Such estimates matche with the actual market excess return over the same sample period. The interaction term continues to be significant, while idiosyncratic volatility is insignificant. Our evidence, once again, suggests that beta estimated based on the past information can't predict the expected returns for stocks with both large betas and high idiosyncratic volatilities, because of the possible over-pricing and instable beta for these stocks in short-run. Our results is also consistent with Kothari, Shanken, Sloan (1995) who argues that the CAPM relation holds in low frequency data. This is because that beta instability and overpricing is less likely to exist in long-run. What is important from our analysis is that for stocks with relatively low idiosyncratic volatilities and small betas, the beta variable predicts the expected return well. We further illustrate this point in the following section.

### **3.4 Cross-sectional Regression with Partial Sample**

Cross-sectional regression results reveal the important role played by the interaction term in affecting the true beta-return relation. As we argue in section 2.1, this is a result of beta instability among stocks with large beta and high idiosyncratic risk. Potentially, empirical results from the last section is also consistent with an alternative hypothesis that idiosyncratic risk has a pervasive pricing effect through affecting the sensitivity of individual stock returns to the market risk. In this section, we adopt a simple strategy to see if this alternate channel has a dominant effect. In particular, we eliminate a small group of stocks with both large beta and high idiosyncratic volatility. If beta instability is the dominant factor, we should expect to see a significant beta-return relation without introducing the interaction term. Otherwise,

the beta variable will remain to be insignificant. Based on the two-way table results in Table 2, we will study two cases. In the first case, we delete four portfolios of stocks with largest idiosyncratic volatilities and betas in the lower right corner along with four portfolios of stocks with either largest beta or largest idiosyncratic volatility next to them. In this case, we retain 70% of the stocks, or over 90% of the market capitalization. In the second case, we push the limit by only deleting the four portfolios in the lower right corner. The remaining stocks counts over 90% of the total number of stocks, or 95% of the market capitalization. Results are reported in Table 4.

Insert Table 4 Approximately here

For the first case in Panel A of Table 4, we see that the beta variable is indeed significant at a 10% level. This is for individual stock. In contrast, when Fama and MacBeth (1978) claim that beta is significant, they did it for portfolio in early sample period with a similar adjusted  $t$  value (see Malkiel and Xu, 2003). When controlling for idiosyncratic volatility, the beta variable is actually significant at a 5% level. As Ang, et. al. (2006) have shown that the realized idiosyncratic volatility is negatively related to future returns, the negative effect seems to be concentrated on stocks with large idiosyncratic volatilities and betas since it is positive and insignificant for our reduced sample as in Model 1. Instead of controlling for idiosyncratic volatility, we can examine the original Fama and French specification in Model 3. Different from Fama and French (1992), the beta variable is significant at a 1% level, although size and book to market is still significant. This result not only suggests that a multi-risk factor model matters, but also makes the Fama and French's (1993) three factor model to be consistent with the cross-sectional evidence. Idiosyncratic volatility is still insignificant in the Fama and French model specification as show in Model 4. In addition, controlling for return reversal and momentum in Model 6 or illiquidity in Model 7 has no affect to the significance of the

beta variable.

To investigate how pervasive beta instability really is, we further investigate the significance of beta when only eliminate 10% of the sample with largest beta and idiosyncratic volatility in the second case. As show in Panel B of Table 4, beta is insignificant when used alone with a p-vale of 16%. However, when used with idiosyncratic volatility, beta is significant at a 10% level. To our surprise, beta is actually significant at a 5% level in the Fama and French (1992) framework. Since beta and size are correlated to some degree, this result may suggest that both the size variable and the beta variable share common noise, which makes both variables significant in the multivariate regression.<sup>8</sup> The beta variable remains very significant when controlling for other popular variables in Model 4 through Model 7. All these results indicate that beta instability among a small group of stocks is more likely to be the cause for the failure of beta in the conventional cross-sectional regressions.

### 3.5 Beta and Idiosyncratic Volatility

Despite the weak explanatory power of the systematic risk measure beta in the cross-sectional regression, the market factor usually is the most important factor in capturing time-series variations of individual stock returns as acknowledged by Fama and French (1996). This means that the failure of the cross-sectional test of the CAPM is unlikely a result of noisy beta estimates. Alternatively, as argued in section 2.1, beta instability could result in good time-series fit despite weak cross-sectional explanatory power. Although we show the existence of beta instability in the last section by examining partial samples, the selection of a partial sample might look arbitrary. Different from noise in the beta estimate, beta instability might be predictable. Therefore, we can also show directly that controlling for the interaction term in restoring the return-beta relation is a direct result of predicting beta instability. In particular,

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<sup>8</sup>Run, Sun, and Xu (2011) discuss the case noise cancellation in a multivariate regression.

we demonstrate in this section, how instable is the short-run beta estimate, and whether such instability can be predicted by idiosyncratic volatility. By doing so, we further show that the significant CAPM relation found in section 3.3 is not a coincidence.

In order to see if beta is instable over time and inherently predictable, we first estimate beta and idiosyncratic volatility each month for each stock using daily returns in Panel A of Table 5. As shown in Model 1, when regressing the current month beta estimate on the last month beta, the estimated coefficient is only 0.24 although very significant. This means that the short-run beta estimates are not very persistent at all with very low autocorrelation. It is such instability that makes the beta measure incapable of explaining the return in the next period. In contrast, we find that the interaction term between beta and idiosyncratic volatility in the last month can significantly predict the movement in beta in the current month. The estimated coefficient is  $-0.389$  and is statistically significant at a 1% level. For stocks with large idiosyncratic volatility, beta seems to drop in short-run. Consequently, future return will drop too.

Insert Table 5 Approximately Here

Perhaps one may argue that beta instability is due to estimation error when using insufficient data. A second exercise is to use daily returns from last quarter (three month) to estimate beta  $Beta_{d,-3}$ . As shown in Panel B of Table 5, when future beta estimate is regressed on such a new beta measure  $Beta_{d,-3}$ , the correspondingly estimated coefficient increases to 0.45, but is still not sufficiently large enough. In other words, there is still substantial beta instability. Similar to results in Panel A of Table 5, the coefficient estimate increases to 0.56 when the interaction term between past beta and idiosyncratic volatility is also included in the regression. This result is consistent with Galai and Masulis's (1976) prediction that beta

drops with the increase in the total volatility, which largely consists of idiosyncratic volatility for individual stocks.

These results further demonstrate that it is the large changes in the beta estimates from period to period that prevent the beta variable to explain the cross-sectional return differences. With our ability to predict such instability in beta estimates using idiosyncratic risk measure, we are able to recover the explanatory power of the systematic risk measure beta in predicting stock returns.

### 3.6 Predicting the Expected Return

The significance of a factor from a cross-sectional regression does not necessarily indicate a large explanatory power for cross-sectional returns. To further assess the power of our model, we compute the predicted returns from a cross-sectional regression for each stock according to Model 1 of Table 3. The choice of Model 1 instead of the complete Model 7 is for illustration purpose, and to eliminate possible contribution from other factors. We then aggregate individual stocks' expected returns into 100 portfolios' expected returns to achieve better visualization. In particular, we first sort stocks into 10 groups according to their rolling beta measure in June each year, stocks in each group are sorted again into 10 groups according to their realized idiosyncratic volatility measure.<sup>9</sup> The time-series average of each portfolio's expected returns is plotted against the average return of each portfolio in Figure 2. For comparison, we also compute the expected returns of individual stocks using the rolling beta and the realized idiosyncratic volatility only as a base model. The corresponding portfolios' expected returns and the average portfolio returns is plotted in Figure 1.

Insert Figure 1 and Figure 2 Approximately Here

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<sup>9</sup>This sorting schedule ensures the maximum spread in expected returns.

In theory, each portfolio should lie on the 45 degree line in the graph. When the expected return is computed from the base model with beta and idiosyncratic volatility only, there is virtually no relation between the expected returns and the average returns as shown in Figure 1. In contrast, when taking into account the interaction term between beta and idiosyncratic volatility to compute the expected return (Model 1 of Table 3), portfolios are now scattered around the 45 degree line as shown in Figure 2. The dramatic difference between the two graphs not only shows the importance of the traditional CAPM model, but also demonstrates the economic significance of including the interactive term in our simple model.

## 4 Robustness

Given the strong results in supporting the CAPM relation, it is important to investigate the robustness of our results. Despite using popular control variables in our analysis, too often we see that results in other studies are challenged when applying different measures or different samples. Therefore, we further investigate our main results using different measures of beta and idiosyncratic volatility, as well as different samples and sample periods.

### 4.1 Alternative Measures of Beta

Noise in the beta estimates may still be significant in affecting the return-beta relation despite the importance of beta instability. Post-ranking portfolio betas  $Beta_p$  are thus commonly used in cross-sectional regressions to reduce the error-in-variables bias as proposed by Fama and French (1992). To a particular stock, its post-ranking beta measure may still vary over time since the stock could belong to a different portfolio over time. However, some of the instability feature might be lost. Moreover, it may conceal important information contained in individual stock betas as pointed out by Ang, Schwarz, and Liu (2010). Since the performance of beta

in our model relies on not only how it captures the instability feature of the beta estimate but also the accuracy of the estimate, we use portfolio beta in Panel A of Table 6 to see if the results still hold.

Insert Table 6 Approximately Here

Again, the overall results are similar to those reported in Panel A of Table 3. As expected, the estimate on  $Beta_p$  increase somewhat although still insignificant when the variable is used alone in the regression (see Model 2). When controlling for the interaction term in Model 1, the portfolio beta becomes significant without the popular control variables. However, the coefficient estimate of 0.9% seems to be too large compared to the average market risk premium of 0.44% over the same time period. Such an estimate does drop significantly to 0.5% after controlling for the size and the book-to-market factors although significant at a 5% level. Further control for return reversal, momentum, and liquidity does not seem to alter the estimate. It is also interesting to see that idiosyncratic volatility alone have positive and significant effect on the expected return once we control for the interaction between idiosyncratic volatility and beta. This suggests that the idiosyncratic volatility puzzle of Ang, et. al. (2006) is limited to stocks with large beta.

In general, volatility can be accurately estimated when using high frequency returns as pointed out by Merton. By the same token, we may be able to estimate beta more accurately when using daily returns. It is also possible that such a beta estimate may be noisier than the rolling monthly estimate when market microstructure effect is important. In balance, we choose to use past three month daily returns to estimate individual stocks' beta measures. Results are reported in Panel B of Table 6. When beta is used alone in Model 2, the result is even worse with a negative and insignificant coefficient estimate. However, our model still

holds when the interaction term is used as shown in Model 1. The coefficient estimate for the beta variable is again close to the market excess return when all the control variables are included in the regression (see Model 5).

## 4.2 Alternative Measures of Idiosyncratic Volatility

Since idiosyncratic volatility plays an important role in our main results, it is important to see if results continue to hold when alternative measures of idiosyncratic volatility is used. In addition to the realized idiosyncratic volatility measure computed using daily residual returns as in Campbell, et. al. and Ang et al (2006), the rolling realized idiosyncratic volatility measure of Bali, et. al. (2009) is an alternative approach.<sup>10</sup> Such a measure is computed using the past 24 to 60 monthly residual returns with respect to the Fama and French's (1993) three factors plus the momentum factor. Results are reported in Table 7.

Insert Table 7 Approximately here

The overall results are very similar to those reported in Table 3. Consistent with Bali, et. al. (2009), idiosyncratic volatility is now only marginally significant as shown in Model 3. What is surprising is that the beta variable is also marginally significant at the same time. When controlling for the interaction term (see Model 1), the beta variable becomes significant as expected. Different from the realized idiosyncratic volatility measure using daily residual returns within a month, the rolling idiosyncratic volatility measure puts more emphasis on intertemporal dependence. It is such intertemporal dependence that helps to predict future beta movement. When controlling for size and book-to-market in equation 5, the coefficient estimate for the beta variable increase to 0.6%, similar to that reported in Table 3. We find

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<sup>10</sup>We do not use Fu's (2009) EGARCH idiosyncratic volatility measure since we are not focusing on the pricing effect of idiosyncratic volatility. As discussed in section 2.1, beta instability is related to the mispricing effect of idiosyncratic volatility instead.

that results are quite closer to our original results after further control for return reversal, momentum, and liquidity in Model 5.

Although theory requires us to use idiosyncratic risk in our empirical study, estimate a measure of idiosyncratic risk is model dependent. In general, idiosyncratic risk counts over 80% of the variations in individual stocks. As an alternative, we can simply use the total volatility as a proxy for idiosyncratic risk as a robust check. Of course, other things being equal, it will bias us against finding supporting evidence when using a noisy measure. However, if we continue to find supporting evidence, our conclusion reached in the empirical section is very robust. Therefore, we replace idiosyncratic volatility in Panel B of Table 7 by total volatility, estimated using daily returns within a month.

Our basic conclusion for the significant beta variable continues to hold well. Although the coefficient estimates for each model in Panel B of Table 7 seems to be a little larger than those in Table 3, they could be biased upward since the total volatility is a noisy measure of the idiosyncratic volatility. We also see that the coefficient estimate for the total volatility is positive but insignificant as shown in Model 1. However, total idiosyncratic volatility becomes negative and remains insignificant after controlling for other popular variables as shown in Models 4 and 5. Finally, the beta variable is always positive and very significant once the interaction term is controlled for.

### **4.3 Subsample**

The beta stability issue might be different for stocks traded on different exchange. Stocks traded on NYSE/AMEX tend to be large mature firms, while NASDAQ firms are usually young firms. Despite the difference, the coefficient estimates should be similar since the CAPM should hold for all stocks. Therefore, we will exam the significance of the beta variable for

the two subsample markets, that is *NYSE/AMEX* market and *NASDAQ* Market. Results are reported in Table 7. The behavior of idiosyncratic volatility has changed a lot in the past decade. In fact, Campbell, Lettau, Malkiel, and Xu (2001) have documented that idiosyncratic risk has been on the rise in recent decade. At the same time, the explanatory power of the market factor becomes less significant in explaining the time-series variation. In order to make sure the robustness of our approach, we further divide our sample period equally into two subsample periods, from 1963 to 1986 and from 1987 to 2010. Table 8 summarizes the main results. For comparability, we use the same measure of beta and idiosyncratic volatility as in Table 3. However, if both the rolling beta and the rolling idiosyncratic volatility measures are used instead, results are much stronger.

Insert Table 8 Approximately here

Comparing estimates in Panel A and Panel B in Table 8, the beta variable is insignificant for both exchange groups when used alone. Perhaps it is more so for the *NASDAQ* group. In the Fama and French's (1992) framework with the added idiosyncratic volatility variable, the book-to-market variable is significant for both groups of stocks. However, the size variable is only marginally significant for *NASDAQ* stocks (Model 3 in Panel B). When controlling for the interaction term in Model 1, the beta variable become significant for both groups of stocks. However, the coefficient estimate is much smaller for *NASDAQ* stocks than that for *NYSE/AMEX* stocks. Finally, with all the control variables, the coefficient estimates for beta are 0.4% and 0.3 for *NYSE/AMEX* and *NASDAQ*, respectively. Therefore, the results are stronger for large and mature firms than for young firms.

When separating the whole sample period into two subsample periods, our basic results for the beta variable continue to hold as shown in Models 1, 4 and 5 of Table 9 for both

subsample periods. Consistent with the evidence on dropping risk premium in recent years, the coefficient estimates for the beta variable are 0.5% and 0.4% for the subsample periods from 1963 to 1986 and from 1987 to 2011, respectively. In addition, it is also interesting to see that the size variable is marginally significant in the first subsample period, but insignificant in the second subsample period, which is consistent with other studies. Therefore, our results are also robust with respect to different subsamples and sample periods.

Insert Table 9 Approximately here

In summary, our results found in Table 3 are neither sensitive to different measures of beta estimate and idiosyncratic volatility, nor sensitive to different subsamples. Therefore, the systematic risk measure beta still matters in differentiate cross-sectional returns of individual stocks.

## 5 Conclusion

In quest for understanding the failure of the CAPM model from an empirical perspective, we offer an alternative story related to the instability of the beta estimate. Even when the CAPM holds period by period in a dynamic setting, for example, the current period beta estimate may not be reliable in predicting future returns when beta changes dramatically from period to period. One of the reasons for large changes in beta is due to investors' behavior of chasing certain stocks. Their action will not only move the stocks they are after, but also move the overall market. This suggests that these stocks' beta estimates will change in short term. Since there are evidence that investors tend to chase stocks with large idiosyncratic volatility, we propose a novel way to control for such change in beta. In particular, we add an interaction between beta and idiosyncratic volatility as a control to the original cross-sectional regression

specification. By doing, we are able to restore the predictive power of beta in cross-sectional regression.

From an empirical perspective, we demonstrate that the beta variable is not only significant in predicting future returns, but the coefficient estimate is very close to the actual risk premium over the same time period as predicted by the CAPM theory. This is only true after controlling for the interaction between beta and idiosyncratic volatility. Our results are very robust in terms of popular controls, different measures of beta and idiosyncratic volatility, different groups of stocks, and subsample periods. In addition, we also show that beta does seem to change a lot from period to period, and much of the change can be predicted by the interaction between beta and idiosyncratic volatility.

Our main contribution lies in not only documenting large changes in beta estimates from period to period, but also offering a practical way to predict such changes in order to restore the CAPM relation. Of course, there might be other ways to control for such changes from both behavior side and rational side. It is our belief that the fundamental CAPM relation holds, at least in the first order, but the relation is likely distorted among a small group of stocks from time to time.

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Table 1: **Summary Statistics**

This table provides the summary statistics for variables used in the study. The sample period spans from July 1963 to December 2010.  $RET$  is the monthly dividend and split-adjusted return.  $Beta_r$  represents an individual stock's rolling beta, estimated from a market model using the past 60-month stock returns.  $Beta_p$  is the portfolio beta, estimated following the procedure in Fama French (1992).  $Beta_{(d,-1)}$  and  $Beta_{(d,-3)}$  are the monthly rolling beta, estimated using a stock's daily returns within last month or within last 3 months.  $IV_d$  is the monthly realized idiosyncratic volatility based on the daily residual stock returns from the past month with respect to the Carhart's four factor model (see Ang et al, 2006).  $IV_r$  is the rolling realized idiosyncratic volatility, computed following Bali and Cakici (2009).  $Ln(ME)$  is the log market capitalization (ME) of the last June, and  $Ln(BM)$  is the log of the fiscal year-end book value of equity divided by the calendar year-end market value of equity.  $RET_{(-2,-7)}$  is the compounded gross return from months  $t - 7$  to  $t - 2$  (inclusive).  $Illiq$  is the last year Amihud illiquidity measure defined in Amihud (2002) and calculated in last year  $y - 1$ .  $P25$  and  $P75$  represent 25% and 75% percentile, respectively. In order to control for the potential data errors and extreme values, all variables are winsorized at the 0.5% and the 99.5% level.

	Mean	STD	P25	Median	P75
$RET$	0.012	0.159	-0.067	0.000	0.075
$Beta_r$	1.160	0.740	0.687	1.090	1.544
$Beta_p$	1.358	0.332	1.139	1.330	1.623
$Beta_{(d,-1)}$	0.737	1.374	0.066	0.666	1.368
$Beta_{(d,-3)}$	0.761	0.868	0.238	0.695	1.229
$IV_d$	0.127	0.108	0.060	0.096	0.157
$IV_r$	0.126	0.072	0.076	0.109	0.157
$TV$	0.151	0.121	0.075	0.118	0.186
$Ln(ME)$	11.515	2.154	9.918	11.362	13.001
$Ln(BM)$	-0.466	0.965	-0.989	-0.379	0.165
$RET_{(-2,-7)}$	0.078	0.431	-0.156	0.027	0.228
$Illiq$	0.056	0.242	0.000	0.002	0.020

Table 2: **Sorted Portfolio Returns**

This table shows the average stock returns of the 25 equal weighted portfolios formed by first sorting individual stocks according to their beta estimates,  $Beta_r$ , estimated using past 60 month returns, or  $Beta_p$ , estimated following Fama and French (1992), and then sorting stocks according to idiosyncratic volatilities,  $IV_d$ , based on the daily residual stock returns from the past month with respect to the Carhart's four factor model (see Ang et al, 2006). The sample period ranges from July 1963 to Dec. 2010.  $H - L$  represents the portfolio return difference between the highest and lowest portfolios. In panel A, computed are equal weighted returns of portfolio sorted by  $Beta_r$  and  $IV_d$ , while in Panel B computed are equal weighted returns of portfolio sorted by  $Beta_p$  and  $IV_d$ . The robust Newey West  $t$ -statistic is reported in the bracket. The symbols \*, \*\*, \*\*\* denote significance level of 10%, 5%, and 1%, respectively.

<b>Panel A: Rolling Beta</b>						
Column{ $Beta_r$ }; Row{ $IV_d$ }						
	1(low)	2	3	4	5(High)	H-L
1(low)	0.89 (5.65)	1.09 (6.11)	1.21 (5.85)	1.23 (5.57)	1.26 (3.82)	0.37 (1.33)
2	1.05 (6.96)	1.25 (6.88)	1.33 (6.92)	1.42 (6.25)	1.29 (4.35)	0.24 (1.04)
3	1.15 (6.31)	1.28 (6.72)	1.37 (6.35)	1.39 (5.54)	1.17 (3.59)	0.02 (0.07)
4	1.13 (5.58)	1.38 (6.42)	1.52 (5.53)	1.23 (4.09)	0.9 (2.43)	-0.23 (-0.82)
5(high)	1.31 (4.77)	1.41 (4.21)	1.38 (3.97)	1.17 (3.16)	0.62 (1.47)	-0.70*** (-2.61)
H-L	0.43* (1.85)	0.32 (1.07)	0.17 (0.58)	-0.06 (-0.23)	-0.64** (-2.24)	
<b>Panel B: Portfolio Beta</b>						
Column{ $Beta_p$ }; Row{ $IV_d$ }						
	1(low)	2	3	4	5	H-L
1(low)	0.92 (6.42)	1.06 (6.79)	1.15 (7.40)	1.17 (6.62)	1.07 (5.70)	0.15 (1.08)
2	1.02 (5.62)	1.22 (6.53)	1.32 (6.47)	1.25 (5.62)	1.06 (3.55)	0.04 (0.20)
3	1.25 (5.95)	1.38 (6.43)	1.41 (6.37)	1.4 (5.31)	1.11 (3.43)	-0.14 (-0.64)
4	1.19 (5.46)	1.47 (5.66)	1.45 (4.96)	1.31 (4.08)	1.01 (2.84)	-0.18 (-0.82)
5	1.45 (5.11)	1.54 (4.58)	1.5 (4.37)	1.09 (2.95)	0.67 (1.54)	-0.78*** (-3.29)
H-L	0.53** (1.99)	0.48 (1.51)	0.36 (1.15)	-0.07 (-0.22)	-0.41 (-1.21)	

Table 3: Fama-MacBeth Regression for Individual Stocks

This table presents the Fama-MacBeth regression results of monthly individual stock returns on different factors. These factors includes,  $Beta_r$ , estimated using last 60 monthly return, idiosyncratic volatility  $IV_d$  estimated based on the daily residual stock returns from the past month with respect to the Carhart's four factor model (see Ang et al, 2006), the log market capitalization  $Ln(ME)$  of the last June, the log of book-to-market  $Ln(BM)$ , last month return  $RET_{(-1)}$ , the compounded gross return from months  $t-7$  to  $t-2$  (inclusive)  $RET_{(-2,-7)}$ , and the Amihud illiquidity measure defined in Amihud (2002)  $Illiq$ .  $Beta_r$ ,  $*IV_d$  is the interaction term between beta and idiosyncratic volatility. In order to control for the potential data errors and extreme values, all variables are winsorized at the 0.5% and the 99.5% level. Monthly returns are dividend and split-adjusted, in percentages. The robust Newey West t-statistic is reported in the bracket. The symbols \*, \*\*, \*\*\* denote significance level at the 10%, 5%, and 1%, respectively.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
$IV_d$	0.024 (1.26)			-0.047*** (-4.98)	-0.005 (-0.37)	0.002 (0.15)	-0.008 (-0.59)
$Beta_r * IV_d$	-0.037*** (-4.3)				-0.034*** (-4.14)	-0.028*** (-3.67)	-0.026*** (-3.45)
$Beta_r$	0.005*** (3.13)	0.001 (0.49)	0.001 (0.67)	0.002 (1.49)	0.006*** (3.64)	0.004*** (2.84)	0.004*** (2.97)
$Ln(BM)$			0.003*** (4.23)	0.002*** (3.78)	0.002*** (4.01)	0.003*** (3.94)	0.002*** (3.71)
$Ln(ME)$			-0.001 (-1.63)	-0.001*** (-3.34)	-0.001*** (-3.27)	-0.001** (-2.56)	-0.001** (-1.98)
$RET_{(-1)}$						-0.065*** (-8.86)	-0.065*** (-8.76)
$RET_{(-2,-7)}$						0.006** (2.51)	0.006** (2.50)
$Illiq$							0.033** (2.53)

Table 4: **Fama-MacBeth Regression for Partial Sample**

This table presents the Fama-MacBeth regression results of monthly individual stock returns on different factors. These factors includes,  $Beta_r$  estimated using last 60 monthly return, idiosyncratic volatility  $IV_d$  estimated based on the daily residual stock returns from the past month with respect to the Carhart's four factor model (see Ang et al, 2006), the log market capitalization  $Ln(ME)$  of the last June, the log of book-to-market  $Ln(BM)$ , last month return  $RET_{(-1)}$ , the compounded gross return from months  $t - 7$  to  $t - 2$  (inclusive)  $RET_{(-2,-7)}$ , and the Amihud illiquidity measure defined in Amihud (2002)  $Illiq$ . In order to control for the potential data errors and extreme values, all variables are winsorized at the 0.5% and the 99.5% level. Monthly returns are dividend and split-adjusted, in percentages. The robust Newey West t-statistic is reported in the bracket. The symbols \*, \*\*, \*\*\* denote significance level at the 10%, 5%, and 1%, respectively.

	Model 1	Model 2	Model 3	Model 4	Model 6	Model 7
<b>Panel A: With 70% of the Sample</b>						
$IV_d$	0.020 (1.12)			-0.004 (-0.40)	0.007 (0.65)	0.003 (0.28)
$Beta_r$	0.002** (1.98)	0.002* (1.85)	0.003*** (2.72)	0.003*** (2.60)	0.003** (2.51)	0.003** (2.56)
$Ln(BM)$			0.002*** (3.02)	0.002*** (3.05)	0.002*** (3.07)	0.002*** (2.96)
$Ln(ME)$			-0.001** (-2.02)	-0.001** (-2.22)	-0.001* (-1.65)	-0.001* (-1.65)
$RET_{(-1)}$					-0.063*** (-9.24)	-0.062*** (-9.19)
$RET_{(-2,-7)}$					0.007*** (3.46)	0.007*** (3.56)
$Illiq$						0.012 (0.86)
<b>Panel B: With 90% of the Sample</b>						
$IV_d$	-0.003 (-0.18)			-0.027*** (-2.97)	-0.014 (-1.48)	-0.023** (-2.48)
$Beta_r$	0.002* (1.61)	0.002 (1.54)	0.003** (2.21)	0.003** (2.21)	0.002** (1.91)	0.002** (2.14)
$Ln(BM)$			0.002*** (3.71)	0.002*** (3.46)	0.002*** (3.49)	0.002*** (3.32)
$Ln(ME)$			-0.001** (-1.96)	-0.001** (-2.27)	-0.001** (-2.14)	-0.001* (-1.77)
$RET_{(-1)}$					-0.064*** (-9.94)	-0.063*** (-9.85)
$RET_{(-2,-7)}$					0.007*** (3.05)	0.007*** (3.08)
$Illiq$						0.032** (2.29)

Table 5: **Relationship between Beta and Idiosyncratic Volatility**

This table presents the Fama-MacBeth regression results from regressing beta on past beta and idiosyncratic volatility. The dependent variable is beta ( $Beta_d$ ), and is estimated using daily stock returns within the month.  $IV_d$  is the idiosyncratic volatility measure estimated based on the daily residual stock returns from the past month with respect to the Carhart's four factor model (see Ang et al, 2006).  $Beta_{(d,-3)}$  is estimated using past three month daily returns.  $Beta_{d,-1} * IV_{d,-1}$  is the interaction term between last period beta and idiosyncratic volatility. The robust Newey West t-statistic is reported in the bracket. The symbols \*, \*\*, \*\*\* denote significance level at the 10%, 5%, and 1%, respectively.

Panel A: 1 Month		
	Model 1	Model 2
$Beta_{d,-1}$	0.242*** (10.35)	0.365*** (11.80)
$Beta_{d,-1} * IV_{d,-1}$		-0.596*** (-5.10)
Panel B: 3 Months		
	Model 1	Model 2
$Beta_{d,-3}$	0.450*** (13.71)	0.560*** (18.17)
$Beta_{d,-3} * IV_{d,-1}$		-0.562*** (-3.53)

Table 6: Cross-sectional Regression with Alternative Beta Measure

This table presents the cross-sectional regression results using alternative beta measure. These measures include the portfolio beta,  $Beta_p$ , estimated following Fama and French's (1992) method, and the rolling beta,  $Beta_{(d,-3)}$ , estimated using past three month daily returns. Other regressors include idiosyncratic volatility  $IV_d$  estimated based on daily residual returns with respect to the Carhart's four factor model (see Ang et al, 2006), the log market capitalization  $Ln(ME)$  of the last June, the log of book-to-market  $Ln(BM)$ , last month return  $RET_{(-1)}$ , the compounded gross return from months  $t - 7$  to  $t - 2$  (inclusive)  $RET_{(-2,-7)}$ , and the monthly Amihud illiquidity measure defined in Amihud (2002)  $Illiq$ .  $Beta * IV_d$  is the interaction term between beta and idiosyncratic volatility. In order to control for the potential data errors and extreme values, all variables are winsorized at the 0.5% and the 99.5% level. Monthly returns are dividend and split-adjusted, in percentages. The robust Newey West t-statistic is reported in the bracket. The symbols \*, \*\*, \*\*\* denote significance level at the 10%, 5%, and 1%, respectively.

	Model 1	Model 2	Model 3	Model 4	Model 5
Panel A: Portfolio Beta					
$IV_d$	0.046** (2.22)		-0.047*** (-4.84)	0.007 (0.46)	0.005 (0.32)
$Beta_p * IV_d$	-0.054*** (-4.45)			-0.038*** (-3.65)	-0.032*** (-3.03)
$Beta_p$	0.009*** (3.03)	0.003 (0.84)	0.002 (1.02)	0.005** (2.24)	0.005** (2.00)
$Ln(BM)$			0.002*** (3.87)	0.002*** (3.99)	0.002*** (3.67)
$Ln(ME)$			-0.001*** (-3.26)	-0.001*** (-3.09)	-0.001* (-1.69)
$RET_{(-1)}$					-0.063*** (-8.64)
$RET_{(-2,-7)}$					0.006** (2.21)
$Illiq$					0.032** (2.50)
Panel B: Individual Beta Using Three Month Daily Returns					
$IV_d$	-0.007 (-0.46)		-0.049*** (-5.21)	-0.031*** (-3.5)	-0.024*** (-2.62)
$Beta_{(d,-3)} * IV_d$	-0.019*** (-3.81)			-0.024*** (-4.72)	-0.021*** (-4.41)
$Beta_{(d,-3)}$	0.003** (1.96)	0.000 (-0.36)	0.001 (1.46)	0.005*** (3.74)	0.004*** (3.07)
$Ln(BM)$			0.002*** (3.79)	0.002*** (3.93)	0.002*** (3.58)
$Ln(ME)$			-0.002*** (-3.1)	-0.002*** (-3.43)	-0.001** (-2.19)
$RET_{(-1)}$					-0.062*** (-8.5)
$RET_{(-2,-7)}$					0.005** (2.15)
$Illiq$					0.033** (2.58)

Table 7: Cross-sectional Regression with Alternative Idiosyncratic Volatility Measure

This table presents the cross-sectional regression results using alternative idiosyncratic volatility measure. The regressors includes,  $Beta_r$ , estimated using the last 60 month return, idiosyncratic volatility  $IV_r$ , estimated based on rolling residual monthly returns, or total volatility  $TV$  estimated based on daily residual return within last month, the log market capitalization  $Lm(ME)$  of the last June, the log of book-to-market  $Lm(BM)$ , last month return  $RET_{(-1)}$ , the compounded gross return from months  $t-7$  to  $t-2$  (inclusive)  $RET_{(-2,-7)}$ , and the monthly Amihud illiquidity measure defined in Amihud (2002)  $Illiq$ .  $Beta_p * IV_d$  is the interaction term between beta and idiosyncratic volatility. In order to control for the potential data errors and extreme values, all variables are winsorized at the 0.5% and the 99.5% level. Monthly returns are dividend and split-adjusted, in percentages. The robust Newey West t-statistic is reported in the bracket. The symbols \*, \*\*, \*\*\* denote significance level at the 10%, 5%, and 1%, respectively.

	Panel A: $IV_r$				Panel B: $TV_d$			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
$IV$	0.05 (1.41)	-0.033* (-1.94)	0.012 (0.42)	-0.006 (-0.21)	0.023 (1.34)	-0.040*** (-4.38)	-0.001 (-0.06)	-0.004 (-0.32)
$Beta_r * IV$	-0.039*** (-3.18)		-0.035*** (-3.02)	-0.028*** (-2.59)	-0.034*** (-4.9)		-0.031*** (-4.81)	-0.025*** (-4.01)
$Beta_r$	0.005** (2.39)	0.002* (1.70)	0.006*** (3.03)	0.005*** (2.95)	0.006*** (3.86)	0.002* (1.74)	0.006*** (4.51)	0.005*** (3.68)
$Lm(BM)$		0.002*** (3.58)	0.002*** (3.77)	0.002*** (3.17)		0.002*** (3.78)	0.002*** (4.02)	0.002*** (3.71)
$Lm(ME)$		-0.001*** (-3.87)	-0.001*** (-3.77)	-0.001*** (-2.78)		-0.001*** (-3.21)	-0.001*** (-3.14)	-0.001* (-1.81)
$RET_{(-1)}$				-0.067*** (-9.17)				-0.065*** (-8.72)
$RET_{(-2,-7)}$				0.007*** (3.05)				0.006*** (2.50)
$Illiq$				0.028** (2.20)				0.032*** (2.50)

Table 8: Cross-sectional Regression in Different Markets

This table presents the Fama-MacBeth regression results for different stock exchanges. These regressors includes,  $Beta_r$ , estimated using the last 60 monthly return, idiosyncratic volatility  $IV_d$  estimated based on the daily residual stock returns from the past month with respect to the Carhart's four factor model (see Ang et al, 2006), the log market capitalization  $Ln(ME)$  of the last June, the log of book-to-market  $Ln(BM)$ , last month return  $RET_{(-1)}$ , the compounded gross return from months  $t-7$  to  $t-2$  (inclusive)  $RET_{(-2,-7)}$ , and the monthly Amihud illiquidity measure defined in Amihud (2002)  $Illiq$ .  $Beta_p * IV_d$  is the interaction term between beta and idiosyncratic volatility. In order to control for the potential data errors and extreme values, all variables are winsorized at the 0.5% and the 99.5% level. Monthly returns are dividend and split-adjusted, in percentages. The robust Newey West t-statistic is reported in the bracket. The symbols \*, \*\*, \*\*\* denote significance level at the 10%, 5%, and 1%, respectively.

	Panel A: NYSE					Panel B: AMEX&NASDAQ				
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 1	Model 2	Model 3	Model 4	Model 5
$IV_d$	0.022 (1.07)		-0.051*** (-5.03)	-0.009 (-0.6)	-0.018 (-1.16)	-0.01 (-0.83)		-0.034*** (-3.83)	-0.017 (-1.46)	-0.020 (-1.52)
$Beta_r * IV_d$	-0.039*** (-3.91)			-0.035*** (-3.69)	-0.025*** (-2.75)	-0.013*** (-2.95)			-0.015*** (-3.03)	-0.010** (-2.06)
$Beta_r$	0.005*** (3.11)	0.001 (0.44)	0.001 (1.27)	0.005*** (3.27)	0.004** (2.51)	0.003** (2.03)	0.00 (0.00)	0.002 (1.36)	0.005*** (2.77)	0.003* (1.87)
$Ln(BM)$			0.002*** (3.28)	0.002*** (3.45)	0.002*** (3.15)			0.004*** (5.49)	0.004*** (5.57)	0.004*** (4.91)
$Ln(ME)$			-0.001*** (-3.06)	-0.001*** (-3.00)	-0.001* (-1.93)			-0.001* (-1.83)	-0.001** (-2.00)	0.000 (-0.73)
$RET_{(-1)}$					-0.058*** (-6.93)					-0.055*** (-9.48)
$RET_{(-2,-7)}$					0.007*** (2.60)					0.001 (0.53)
$Illiq$					0.040*** (2.99)					0.019*** (2.77)

Table 9: Cross-sectional Regression Results over Different Subsample Periods

This table presents the Fama-MacBeth regression results over different subsample periods. These regressors includes,  $Beta_r$ , estimated using the last 60 monthly return, idiosyncratic volatility  $IV_t$  estimated based on the daily residual stock returns from the past month with respect to the Carhart's four factor model (see Ang et al, 2006), the log market capitalization  $Ln(ME)$  of the last June, the log of book-to-market  $Ln(BM)$ , last month return  $RET_{(-1)}$ , the compounded gross return from months  $t-7$  to  $t-2$  (inclusive)  $RET_{(-2,-7)}$ , and the monthly Amihud illiquidity measure defined in Amihud (2002)  $Illiq$ .  $Beta_p * IV_t$  is the interaction term between beta and idiosyncratic volatility. In order to control for the potential data errors and extreme values, all variables are winsorized at the 0.5% and the 99.5% level. Monthly returns are dividend and split-adjusted, in percentages. The robust Newey West t-statistic is reported in the bracket. The symbols \*, \*\*, \*\*\* denote significance level at the 10%, 5%, and 1%, respectively.

	Panel A: 1963-1986					Panel B: 1987-2010				
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 1	Model 2	Model 3	Model 4	Model 5
$IV_t$	0.048 (1.40)		-0.067*** (-4.87)	0.002 (0.10)	0.004 (0.16)	0.001 (0.06)		-0.027*** (-2.95)	-0.012 (-0.94)	-0.019 (-1.48)
$Beta_p * IV_t$	-0.060*** (-4.51)			-0.054*** (-4.17)	-0.045*** (-3.66)	-0.015*** (-3.13)			-0.014*** (-2.9)	-0.008* (-1.84)
$Beta_p$	0.007** (2.47)	0.000 (0.03)	0.001 (0.58)	0.006** (2.53)	0.005** (2.21)	0.004** (1.98)	0.001 (0.75)	0.002 (1.60)	0.005*** (2.69)	0.004** (2.00)
$Ln(BM)$			0.002** (1.98)	0.002** (2.16)	0.002** (2.13)			0.003*** (3.63)	0.003*** (3.76)	0.003*** (3.31)
$Ln(ME)$			-0.002*** (-2.67)	-0.002** (-2.55)	-0.001* (-1.67)			-0.001** (-2.42)	-0.001** (-2.46)	-0.001 (-1.2)
$RET_{(-1)}$					-0.081*** (-7.5)					-0.049*** (-7.77)
$RET_{(-2,-7)}$					0.010*** (3.47)					0.002 (0.63)
$Illiq$					0.041* (1.66)					0.025*** (3.13)

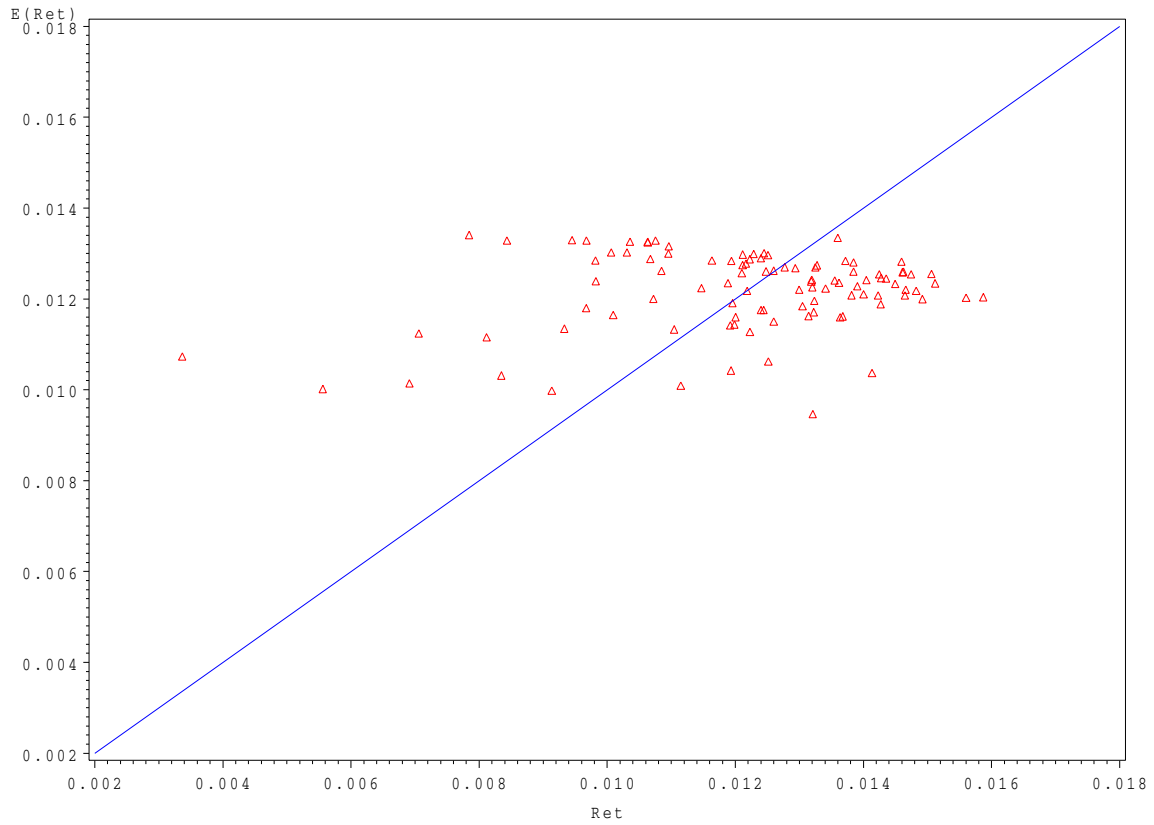
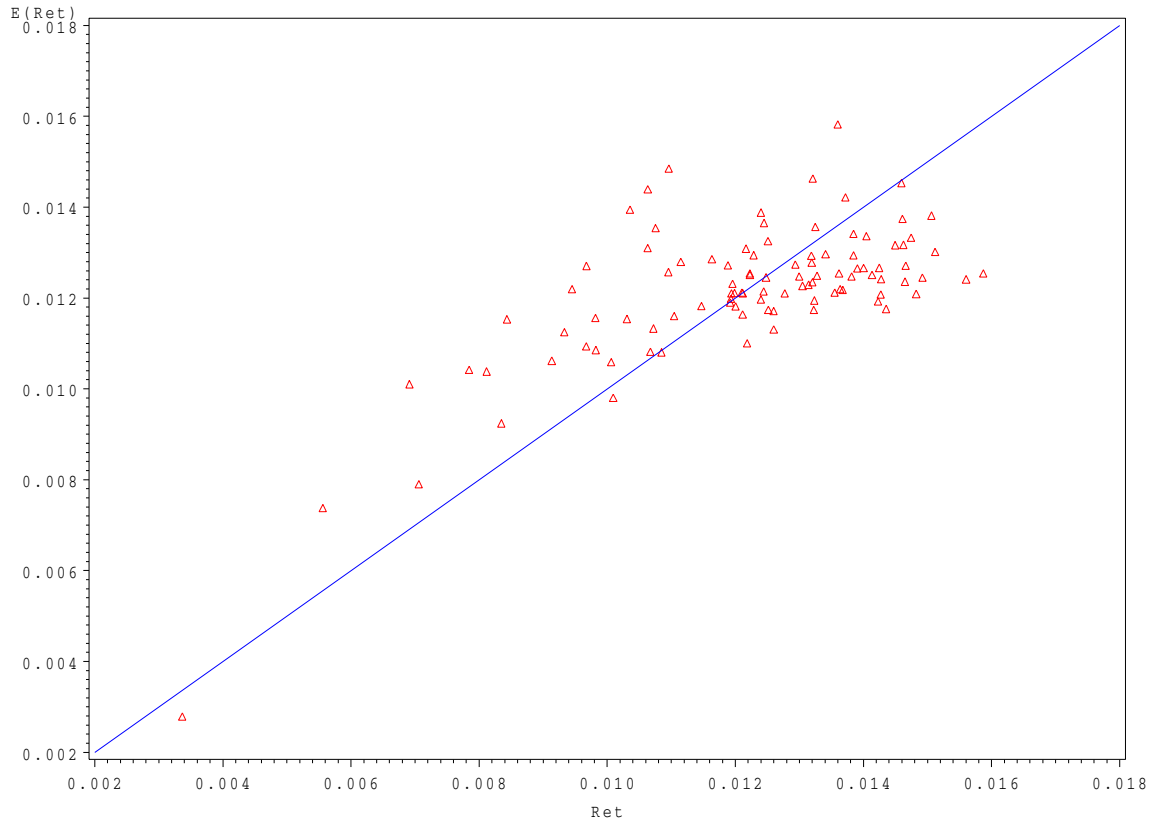


Figure 1: **Realized vs. Expected Return Estimated Using Beta and Idiosyncratic Volatility**

This graph presents the relation between the realized return and expected return of 100 beta-idiosyncratic volatility sorted portfolio. The expected return is estimated from cross-sectional regressions using rolling beta estimates and the realized idiosyncratic volatilities of individual stocks only. The expected return is on the  $y$  axis and the realized return is on the  $x$  axis.



**Figure 2: Realized vs. Expected Return Estimated Using Beta, Idiosyncratic Volatility and Interaction Term**

This graph presents the relation between the realized return and expected return of 100 beta-idiosyncratic volatility sorted portfolio. The expected return is estimated from cross-sectional regressions using rolling beta estimates, the realized idiosyncratic volatilities, and the interaction between beta and idiosyncratic volatility of individual stocks. The expected return is on the  $y$  axis and the realized return is on the  $x$  axis.