

Market Timing, Investment, and Risk Management*

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Abstract

Firms face uncertain financing conditions and are exposed to the risk of a sudden rise in financing costs during financial crises. We develop a tractable model of dynamic corporate financial management (cash accumulation, investment, equity issuance, risk management, and payout policies) for a financially constrained firm facing time-varying external financing costs. Firms value financial slack and build cash reserves to mitigate financial constraints. However, uncertainty about future financing opportunities also induce firms to rationally time the equity market, even if they have no immediate needs for cash. The stochastic financing conditions have rich implications for investment and risk management: (1) investment can be *decreasing* in financial slack; (2) firms may invest *less* as expected future financing costs fall; (3) investment-cash sensitivity, marginal value of cash, and firm's risk premium can all be non-monotonic in cash holdings; (4) speculation (as opposed to hedging) can be value-maximizing for financially constrained firms.

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1 Introduction

The financial crisis of 2008 is a fresh reminder of the substantial uncertainties about financing conditions that corporations face at times, as well as the impact that market shutdowns can have on the economy. Recent studies have documented dramatic changes in firms' financing and investment behaviors during the crisis. For example, Ivashina and Scharfstein (2009) document aggressive credit line drawdowns by firms for precautionary reasons. Campello, Graham, and Harvey (2009) and Campello, Giambona, Graham, and Harvey (2010) show that the financially constrained firms planned deeper cuts in investment, spending, burned more cash, drew more credit from banks, and also engaged in more asset sales in the crisis.

Intuitively it is quite sensible that firms should try to adapt to the fluctuations in financing conditions, including timing favorable market conditions and hedging against unfavorable market conditions. However, there is little existing theoretical work that tries to answer the following questions: How should firms change their financing, investment, and risk management policies during a period of severe financial constraints? And how should firms behave when facing the threat of financial crisis in the future?

In this paper we address the above questions by proposing a dynamic model of investment, financing, and risk management for firms facing stochastic financing conditions. Our model combines the corporate precautionary cash saving motive due to financial constraints, developed in Bolton, Chen, and Wang (2010) (henceforth BCW), with the market timing motives that endogenously arise due to stochastic financing opportunities. The four main building blocks of the model are: 1) a long-run constant-returns-to-scale production function with independently and identically distributed (i.i.d.) productivity shocks, convex investment adjustment costs, and a constant capital depreciation rate (as in Hayashi (1982)); 2) stochastic external equity financing costs; 3) constant cash carry costs; and 4) dynamic hedging opportunities. We purposely hold the investment opportunities constant in order to highlight the role of time-varying financing conditions.

We analyze how a firm simultaneously adjusts its cash reserves, investment, hedging,

financing, and payout decisions in two settings. In one case, the firm is in the midst of a financial crisis trying to survive so as to preserve the firm’s going-concern value. In a second case, we consider a firm currently facing relatively favorable financing conditions, but is anticipating a potential financial crisis that will freeze up financial markets.

The main results of our model are as follows. First, during a period of high external financing costs (e.g., a financial crisis), the firm cuts investment and delays payout aggressively in order to survive the crisis.¹ While in general, the sooner the crisis is expected to end, the less valuable cash can be to mitigate financial constraints, we show that the opposite can be true when cash holding is low. The intuition for this seemingly counter-intuitive result is as follows. With low cash holding, the firm is facing an immediate liquidation threat. When the crisis is expected to end soon, the “breathing room” provided by an extra dollar of cash becomes especially valuable. This effect can cause the marginal value of cash to rise as the expected duration of the crisis gets shorter. It can also cause firms with low cash holdings to underinvest more aggressively while its expected future financing costs are falling, whereas firms with relatively high cash holdings will invest more at the same time. Another interesting finding in the crisis state is that the firm’s payout boundary is first increasing and then decreasing in the probability of exiting the crisis.

Second, we show that it may be optimal for firms to time equity markets. When there is a significant chance that financing conditions will deteriorate dramatically, the firm will optimally time the market by issuing new equity before it runs out of cash. Otherwise, the window of opportunity for cheap equity funding may vanish. The timing results are consistent with the findings in Baker and Wurgler (2002), DeAngelo, DeAngelo, and Stulz (2009), Fama and French (2005), and Huang and Ritter (2009).

Moreover, we show that market timing together with fixed costs of external financing can give rise to convexity of firm value for low levels of cash holdings in states with good financing opportunities. The convexity result has several important implications. It implies that investment can be *decreasing* in cash holding simply due to the market timing option.

¹See the empirical evidence cited in the opening paragraph.

This prediction is opposite to most models of investment with financial constraints. It also implies that the risk premium of a financially constrained firm might not necessarily decrease with its cash holding as often perceived. Finally, it implies that speculation instead of risk management can sometimes be value-maximizing for a financially constrained firm.

Our third result is that the firm's risk premium can be decomposed into two parts: a technology risk premium and a financing risk premium. Both components are sensitive to changes in the firm's cash holding, especially in the state of poor financing conditions, where the conditional risk premium ranges from 2% to 30% depending on the firm's cash holding. Moreover, while the technology risk premium generally decreases with cash holding, the market timing effect can make it increase with cash due to the convexity of firm value in cash.

Fourth, as the expected duration of the state with favorable financing conditions shortens, the firm issues equity sooner in that state because the window of opportunity is smaller, and the firm optimally delays cash payouts to shareholders more. Overall, the firm's cash inventory rises in anticipation of a significant worsening of equity financing opportunities. These results confirm the conjecture of Bates, Kahle, and Stulz (2009), who find that the average cash-to-asset ratio of US firms has nearly doubled in the past quarter century, and who attribute this rise in cash holdings to firms' perceived increase in risk. These results also help explain the investment and financing policies of many US non-financial firms in the years prior to the financial crisis of 2007-2008, to the extent that these firms had anticipated a potential worsening of financing conditions.

Our results highlight the sophisticated dynamic interactions between firm savings and investment. Typically, we expect that higher cash holdings or lower expected future financing costs will relax a firm's financial constraint. Hence, investment should increase with cash (and other financial slack measures such as credit) and decrease with expected financing costs. This is generally true and holds in dynamic corporate finance models and also optimal dynamic contracting models in the absence of stochastic financing conditions.² However, we

²See DeMarzo, Fishman, He, and Wang (2010) for an example.

show that with stochastic financing opportunities, investment is no longer monotonically increasing in cash, nor is it monotonically decreasing with expected financing costs. The key to these relations lies in the optionality of market timing and the dynamic behavior of the marginal value of cash.

Our result also shows that first-generation static models on financial constraints and corporate investment³ are inadequate to explain corporate investment policy based on simple comparative statics analysis. In particular, static models are unsuited to explain the effects of market timing on corporate investment, since these effects do not simply operate through a change in the cost of external equity financing or a change in the firm's cash holdings. Rather, market timing matters when there is a *finitely-lived window of opportunity* for cheap equity financing. Moreover, market timing interacts in a complex way with the firm's precautionary cash management: when cash is tight and dwindling it induces an *acceleration* in capital expenditure, while when cash is abundant it induces a deceleration of investment in response to a local reduction in cash holdings.

By construction, the productivity shocks in our model are *i.i.d.* Thus, firms that time equity markets in our model are also ones with low cash holdings (as opposed to having better investment opportunities). This is consistent with the empirical findings of DeAngelo, DeAngelo and Stulz (2009) that most firms who issue stock look as if they are cash constrained. Therefore, one cannot reject the market timing hypothesis based on this finding alone. Certainly, firms may issue equity in good times to finance investment opportunities, but our model shows that firms issuing equity when cash holdings are low can be consistent with a rational market timing explanation. Testing of our market timing hypothesis would ideally look for firm behavior not only in equity issuance, but also in investment and hedging decisions. For cash-strapped firms, corporate investment may increase, and speculation may arise as the firm's cash dwindles and gets closer to the issuance boundary to replenish its cash holding.

To the best of our knowledge, this paper provides the first dynamic model of corporate

³See Froot, Scharfstein and Stein (1993) and Kaplan and Zingales (1997).

investment with stochastic financing conditions. We echo the view expressed in Baker (2010) that supply effects may be significant for corporate finance. While we treat changes in financing conditions as exogenous in this paper, the cause could be time variations in the frictions of financial intermediation, investors' risk aversion, or aggregate uncertainty and information asymmetry. Earlier theoretical work on investment with financial constraints mostly focus on the demand side, i.e., the firm's optimizing behavior taking the financing conditions as constant and time invariant. See Kaplan and Zingales (1997), Gomes (2001), Almeida, Campello, and Weisbach (2004), Hennessy and Whited (2005, 2007), Gamba and Triantis (2008), Riddick and Whited (2009), Bolton, Chen, and Wang (2010), among others.

2 The Model

We build on BCW by introducing stochastic investment and external financing conditions into a firm's dynamic investment, financing, cash management, and hedging problem. Specifically, we assume that the firm can be in one of two states, denoted by $s_t = 1, 2$. In each state, the firm faces different financing and investment opportunities. The state switches from 1 to 2 (or from 2 to 1) over a short time interval Δ with a constant probability $\zeta_1\Delta$ (or $\zeta_2\Delta$). For an analysis with a more general setup, see the appendix.

2.1 Production technology

The firm employs capital as the factor of production and the price of capital is normalized to one. We denote by K and I respectively the firm's capital stock and gross investment. As is standard in capital accumulation models, the capital stock K evolves according to:

$$dK_t = (I_t - \delta K_t) dt, \quad t \geq 0, \quad (1)$$

where $\delta \geq 0$ is the rate of depreciation.

The firm's operating revenue at time t is proportional to its capital stock K_t , and is given

by $K_t dA_t$, where dA_t is the firm's productivity shock over time increment dt . We assume that

$$dA_t = \mu(s_t) dt + \sigma(s_t) dZ_t^A, \quad (2)$$

where Z_t^A is a standard Brownian motion and $\mu(s_t)$ and $\sigma(s_t)$ denote the expected return on capital and its volatility in state s_t . The firm's incremental operating profit dY_t over time increment dt is then given by:

$$dY_t = K_t dA_t - I_t dt - \Gamma(I_t, K_t, s_t) dt, \quad t \geq 0, \quad (3)$$

where $I_t dt$ is the investment over time dt and $\Gamma(I_t, K_t, s_t) dt$ is the additional adjustment cost that the firm incurs in the investment process. Note that we allow the adjustment costs to be state dependent. Following the neoclassical investment literature (Hayashi (1982)), we assume that the firm's adjustment cost is homogeneous of degree one in I and K . In other words, the adjustment cost takes the homogeneous form $\Gamma(I, K, s) = g_s(i)K$, where i is the firm's investment capital ratio ($i = I/K$), and $g_s(i)$ is a state-dependent function that is increasing and convex in i .⁴ Our analysis does not depend on the specific functional form of $g_s(i)$ and to simplify the analysis we assume that $g_s(i)$ is quadratic:

$$g_s(i) = \frac{\theta_s(i - \nu_s)^2}{2}, \quad (4)$$

where θ_s is the adjustment cost parameter and ν_s is a constant parameter.⁵

The firm can liquidate its assets at any time. The liquidation value L_t is proportional to the firm's capital at time t , but the liquidation value per unit of capital can change with the state s_t , that is, $L_t = l_s K_t$, where l_s is the recovery value per unit of capital in state s .

⁴For notational convenience we use the notation x_s to denote a state dependent variable $x(s)$ whenever there is no ambiguity.

⁵In the literature, common choices of ν_s are either zero or the rate of depreciation δ . While the former choice implies zero adjustment cost for zero gross investment, the latter choice implies a zero adjustment cost when net investment is zero.

2.2 Stochastic Financing Opportunities

Neoclassical investment models (Hayashi (1982)) assume that the firm faces frictionless capital markets and that the Modigliani and Miller theorem holds. However, in reality, firms face important financing frictions for incentive, information asymmetry, and transaction cost reasons.⁶ Our model incorporates a number of financing costs that firms face in practice and that empirical research has identified, while retaining an analytically tractable setting.

The firm may choose to use external financing at any point in time. For simplicity, we only consider external equity financing as the source of external funds for the firm. We leave the generalization of allowing the firm to also issue debt for future research. The firm incurs a fixed and a variable cost of issuing external equity. The fixed cost is given by $\phi_s K$, where ϕ_s is the fixed cost parameter in state s . As in BCW we take the fixed cost to be proportional to the firm's capital stock K . This assumption ensures that the firm does not *grow out of its fixed issuing costs*. It is also analytically convenient, as it preserves the homogeneity of the model in the firm's capital stock K . The firm also incurs a (state dependent) proportional issuance cost γ_s for each unit of external funds it raises. That is, after paying the fixed cost $\phi_s K$, the firm pays $\gamma_s > 0$ in state s for each incremental dollar it raises.

We denote by:

1. H the process for the firm's cumulative external financing (so that dH_t is the incremental external financing over time dt);
2. X the firm's cumulative issuance costs;
3. W the process for the firm's cash stock;
4. U the firm's cumulative non-decreasing payout process to shareholders (so that dU_t is the incremental payout over time dt).

Distributing cash to shareholders may take the form of a special dividend or a share

⁶See Jensen and Meckling (1976), Leland and Pyle (1977), and Myers and Majluf (1984), for example.

repurchase.⁷ The benefit of a payout is that shareholders can invest the proceeds at the market rate of return and avoid paying a *carry cost* on the firm’s retained cash holdings. We denote the unit cost of carrying cash inside the firm per unit of time by $\lambda \geq 0$.⁸

If the firm runs out of cash ($W_t = 0$) it needs to raise external funds to continue operating or its assets will be liquidated. If the firm chooses to raise new external funds to continue operating, it must pay the financing costs specified above. The firm may prefer liquidation if the cost of financing is too high relative to the continuation value (e.g., when the firm is not productive, i.e., low μ). We denote by τ the firm’s stochastic liquidation time. Note that $\tau = \infty$ means that the firm never chooses to liquidate.

We may write the dynamics for the firm’s cash W as follows:

$$dW_t = [K_t dA_t - I_t dt - \Gamma(I_t, K_t, s_t)] dt + (r(s_t) - \lambda) W_t dt + dH_t - dU_t. \quad (5)$$

where the first term is the firm’s cash flows from operations dY_t given in (3), the second term is the return (net of the carry cost λ) on W_t , the third term dH_t is the cash inflow from external financing, and the last term dU_t is the cash outflow to investors, so that $(dH_t - dU_t)$ is the net cash flow from financing. Note that this is a completely general financial accounting equation, where dH_t and dU_t are endogenously determined by the firm.

The homogeneity assumptions embedded in the adjustment cost, the “AK” production technology, and financing costs allow us to deliver our key results in a parsimonious and analytically tractable homogeneous model. Adjustment costs may not always be convex and the production technology may exhibit long-run decreasing returns to scale in practice, but

⁷We cannot distinguish between a special dividend and a share repurchase, as we exclude taxes. Note, however, that a commitment to regular dividend payments is suboptimal in our model. We also exclude any fixed or variable payout costs so as not to overburden the model. These can be added to the analysis

⁸The cost of carrying cash may arise from an agency problem or from tax distortions. Cash retentions are tax disadvantaged because the associated tax rates generally exceed those on interest income (Graham (2000)). Since there is a cost of hoarding cash λ the firm may find it optimal to distribute cash back to shareholders when its cash inventory grows too large. If $\lambda = 0$ the firm has no incentives to pay out cash since keeping cash inside the firm does not have any disadvantages, but still has the benefit of relaxing financial constraints. We could also imagine that there are settings in which $\lambda \leq 0$. For example, if the firm may have better investment opportunities than investors. We do not explore this case in this paper as we are interested in a trade-off model for cash holdings.

these functional forms substantially complicate the formal analysis.⁹ As will become clear below, the homogeneity of our model in W and K allows us to reduce the dynamics to a one-dimensional equation, which is relatively straightforward to solve.

2.3 Systematic Risk and the Pricing of Risk

There are two different sources of systematic risks in our model: *i*) a small, continuous, diffusion shock, and *ii*) a large discrete shock when the economy switches from one state of nature to another. The diffusion shock in any given state s may be correlated with the firm's productivity shock, and we denote the correlation coefficient by ρ . The discrete shock affects both the firm's productivity and its external financing costs, as we have highlighted above.

How are these sources of systematic risk priced? Our model can allow for either risk-neutral or risk-averse investors. If investors are risk neutral, then the pricing of risk is zero and the physical probability distribution coincides with the risk-neutral probability distribution. If investors are risk-averse, however, we need to distinguish between physical and risk-neutral measures. We do so as follows.

For the diffusion risk, we assume that there is a constant market price of risk η_s in each state s . The firm's *risk adjusted* productivity shock (under the risk-neutral probability measure \mathbb{Q}) is then given by

$$dA_t = \hat{\mu}(s_t) dt + \sigma(s_t) d\hat{Z}_t^A, \quad (6)$$

where the mean productivity shock is adjusted to account for the firm's exposure to diffusion risk as follows:

$$\hat{\mu}(s_t) \equiv \hat{\mu}_s = \mu_s - \rho\eta_s\sigma_s,$$

and \hat{Z}_t^A is a standard Brownian motion under the risk-neutral probability measure \mathbb{Q} .¹⁰

⁹See Hennessy and Whited (2005, 2007) for an analysis of a non-homogenous model.

¹⁰In the appendix, we provide a more detailed discussion of systematic risk premia. The key observation

A risk-averse investor also requires a risk premium to compensate for the risk of the economy switching states. As we show in the appendix, this involves transforming the transition intensity under the physical probability measure to the risk-neutral probability measure \mathbb{Q} as follows: let $\hat{\zeta}_1$ and $\hat{\zeta}_2$ denote the transition intensities from respectively state 1 to state 2 (and state 2 to state 1) under the risk-neutral measure, then these intensities are related to their physical intensities as follows:

$$\hat{\zeta}_1 = e^{\kappa_1} \zeta_1, \text{ and } \hat{\zeta}_2 = e^{\kappa_2} \zeta_2,$$

where $\kappa_1 = \ln(\hat{\zeta}_1/\zeta_1)$ and $\kappa_2 = \ln(\hat{\zeta}_2/\zeta_2)$ represent a form of risk premium required by a risk-averse investor for the exposure to this jump risk.

Note that a positive κ_s implies that $\hat{\zeta}_s > \zeta_s$. In other words, when κ_s is positive it is as if a risk-averse investor perceived a higher transition intensity under the risk-neutral probability measure than under the physical measure. Vice versa, a negative κ_s implies that $\hat{\zeta}_s < \zeta_s$. In other words, the perceived transition intensity for a risk-averse investor under the risk-neutral measure is lower. As we show in the appendix, κ_s is positive in one state and negative in the other. Intuitively, this reflects the idea that a risk-averse investor makes an *upward adjustment* of the transition intensity *from the good to the bad state* (with $\kappa_s > 0$) and a *downward adjustment* of the transition intensity *from the bad to the good state* (with $\kappa_s < 0$). In sum, it is as if a risk-averse investor were uniformly more ‘pessimistic’ than a risk-neutral investor: she thinks ‘good times’ are likely to last shorter and ‘bad times’ longer.

2.4 Firm optimality

The firm chooses its investment I , cumulative payout policy U , cumulative external financing H , and liquidation time τ to maximize firm value as follows:

$$\mathbb{E}_0^{\mathbb{Q}} \left[\int_0^{\tau} e^{-\int_0^t r_u du} (dU_t - dH_t - dX_t) + e^{-\int_0^{\tau} r_u du} (L_{\tau} + W_{\tau}) \right], \quad (7)$$

is that the adjustment from the physical to the risk-neutral probability measure reflects a representative risk-averse investor’s stochastic discount factor (SDF) in a dynamic asset-pricing model.

where r_u denotes the interest rate at time u . The first term is the discounted value of payouts to shareholders, and the second term is the discounted value upon liquidation. Note that optimality may imply that the firm never liquidates. In that case, we simply have $\tau = \infty$.

3 Model Solution

3.1 First-best Benchmark

We begin by characterizing the solution in the neoclassical benchmark, where there are no external financing costs, $\phi_s = \gamma_s = 0$. In the neoclassical (frictionless-markets) solution firms hold no cash ($W = 0$) and the optimal investment is determined by Tobin's q , which is the ratio of the market value and replacement value of capital. As Hayashi (1982) has first established, marginal q is equal to average (Tobin's) q in the first-best benchmark due to the homogeneity in K of the production and adjustment-cost functions.

The first-best Tobin's q and investment-capital ratio i_s^{FB} satisfy

$$r_s q_s^{FB} = \hat{\mu}_s - i_s^{FB} - \frac{1}{2} \theta_s (i_s^{FB} - \nu_s)^2 + q_s^{FB} (i_s^{FB} - \delta) + \hat{\zeta}_s (q_{s^-}^{FB} - q_s^{FB}), \quad s = 1, 2 \quad (8)$$

and

$$q_s^{FB} = 1 + \theta_s (i_s^{FB} - \nu_s). \quad (9)$$

Note first that Tobin's q is greater than one only due to the presence of investment adjustment cost. Second, as described in the system of equations (8), firm value in the first-best benchmark, q_s^{FB} in state s (normalized by the firm's capital stock K), is the sum of the present value of expected earnings net of investment and adjustment costs per unit of capital (under the risk-neutral measure \mathbb{Q}), $\hat{\mu}_s - i_s^{FB} - \frac{1}{2} \theta_s (i_s^{FB} - \nu_s)^2$, plus the value of the net percentage increase in capital stock, $q_s^{FB} (i_s^{FB} - \delta)$, plus the expected change in value (also under \mathbb{Q}) as the firm switches from state s to s^- , $\hat{\zeta}_s (q_{s^-}^{FB} - q_s^{FB})$. In the two-state model, i_s^{FB} and q_s^{FB} can be solved in closed form by mapping this system of bi-variate quadratic

equations into a quartic equation.

3.2 Second-best Solution

Let $P(K, W, s)$ denote firm value when the firm faces positive external financing costs ($\phi_s > 0$ and $\gamma_s \geq 0$) in state s , with capital K and cash holding W . Firm value $P(K, W, s)$ then satisfies the following system of Hamilton-Jacobi-Bellman (HJB) equations when its cash holding is above the financing-liquidation boundary \underline{W}_s and below the payout boundary \overline{W}_s , i.e., for $\underline{W}_s \leq W \leq \overline{W}_s$,

$$\begin{aligned} r_s P(K, W, s) = \max_I [(r_s - \lambda_s) W + \hat{\mu}_s K - I - \Gamma(I, K, s)] P_W(K, W, s) + \frac{\sigma_s^2 K^2}{2} P_{WW}(K, W, s) \\ + (I - \delta K) P_K(K, W, s) + \hat{\zeta}_s (P(K, W, s^-) - P(K, W, s)) \end{aligned} \quad (10)$$

where s^- denotes the other state.

Intuitively, the first and the second terms on the right side of the HJB equation (10) give the effects of the expected change (drift) and volatility of cash holding W on firm value, respectively. The third term gives the effect of the expected change of capital stock K on firm value. The last term gives the expected change of firm value due to the change of the state from s to s^- . When $\hat{\zeta}_s = 0$, we uncover the special case where the firm remains forever in the same state (the case treated in BCW).

As in BCW, firm value is homogeneous of degree one in W and K within each state. We may write $P(K, W, s) = p_s(w)K$, and substitute it into (10) and simplifying, we then obtain the following system of ordinary differential equations (ODE) for $p_s(w)$:

$$\begin{aligned} r_s p_s(w) = \max_{i_s} [(r_s - \lambda_s) w + \hat{\mu}_s - i_s - g_s(i_s)] p'_s(w) + \frac{\sigma_s^2}{2} p''_s(w) \\ + (i_s - \delta) (p_s(w) - w p'_s(w)) + \hat{\zeta}_s (p_{s^-}(w) - p_s(w)). \end{aligned} \quad (11)$$

The first-order condition (FOC) for the investment-capital ratio $i(w)$ is then given by:

$$i_s(w) = \frac{1}{\theta_s} \left(\frac{p_s(w)}{p'_s(w)} - w - 1 \right) + \nu_s, \quad (12)$$

where $p'_s(w)$ is the *marginal value of cash* in state s .

The implied investment response to changes in w is thus given by:

$$i'_s(w) = -\frac{1}{\theta_s} \frac{p_s(w)p''_s(w)}{p'_s(w)^2}. \quad (13)$$

As in BCW, the endogenous payout boundary $\bar{w}_s = \bar{W}_s/K$ satisfies the following value matching condition:

$$p'_s(\bar{w}_s) = 1, \quad (14)$$

which states that the marginal value of cash is one when the firm chooses to pay out cash. Moreover, the optimality of a payout implies the following super contact condition (see, e.g., Dumas, 1991) holds:

$$p''_s(\bar{w}_s) = 0. \quad (15)$$

In contrast, the lower endogenous financing boundary in state s is determined by a fundamentally different trade-off than in the single-state model in BCW. Let $\underline{w}_s = \underline{W}_s/K$ denote the endogenous lower boundary for equity issuance in state s , and let m_s denote the “return target” financing level in state s per unit of capital. A key result in BCW is that the firm never chooses to raise external equity before it exhausts its cash stock. That is, in BCW the firm optimally chooses $\underline{w} = 0$. The reason is that the firm always has the option to raise external equity financing in the future, and market financing terms do not change over time (i.e., financing opportunities are constant). The firm is therefore better off relying first on its cheaper internal funds before turning to external financing. As is highlighted in BCW, this is a form of *dynamic pecking order* of financing.

When financing opportunities are changing, however, as they are in our setting here, it is no longer necessarily optimal to set $\underline{w} = 0$. It may now be optimal for the firm to *time*

the market and issue equity before it runs out of cash, if it is concerned that financing costs could rise in the future. That is, the option to tap cheaper equity markets now even though the firm has not run out of cash can be an optimal strategy if the cheap financing terms are not permanent.

Given any equity issuance boundary \underline{w}_s , however, we have the same value matching and smooth pasting conditions at issuance as in BCW. These allow us, in particular, to determine the return target m_s :

$$p_s(\underline{w}_s) = p_s(m_s) - \phi_s - (1 + \gamma_s)(m_s - \underline{w}_s), \quad (16)$$

$$p'_s(m_s) = 1 + \gamma_s. \quad (17)$$

If the firm chooses to raise external equity, it first pays the fixed equity issuance cost ϕ_s per unit of capital and then incurs the marginal issuance cost γ_s for each unit of equity it raises. The condition (16) thus gives the accounting relation for firm value immediately before and after issuance. Second, as the firm optimally chooses its external financing at the margin it sets m_s so that marginal benefit of issuance $p'_s(m_s)$ is equal to the marginal cost $1 + \gamma_s$, which yields condition (17).

How does the firm determine its equity issuance boundary \underline{w}_s ? We use the following two-step procedure. First, suppose that the optimal lower boundary \underline{w}_s is interior ($\underline{w}_s > 0$), then, the standard optimality condition implies that the derivatives of the left and the right sides of (16) with respect to \underline{w}_s should be equal. This argument gives the following condition:

$$p'_s(\underline{w}_s) = 1 + \gamma_s. \quad (18)$$

If there exists no \underline{w}_s such that the above condition holds, we obtain a corner solution, $\underline{w}_s = 0$. In that case, the option value to tap equity markets earlier than absolutely necessary is valued at zero. Using this procedure, we can characterize the optimal lower boundary $\underline{w}_s \geq 0$.

Next, we need to determine whether costly external equity issuance or liquidation is optimal, as the firm always has the option to liquidate. Under our assumptions, the firm's

capital is productive and thus its going-concern value is higher than its liquidation value. Therefore, the firm never chooses to exercise its liquidation option before it runs out cash. Under liquidation, we then have

$$p_s(0) = l_s. \quad (19)$$

Hence, the firm chooses costly equity issuance as long as the equilibrium firm value $p_s(0)$ is greater than l_s .

Finally, we specify the value function outside of the financing and payout boundary. If the firm has too much cash in state s (so that $w > \bar{w}_s$) it will reduce its cash holding to \bar{w}_s immediately by making a lump-sum payout. That is, we have

$$p_s(w) = p_s(\bar{w}_s) + (w - \bar{w}_s), \quad w > \bar{w}_s. \quad (20)$$

This scenario is possible when the firm with high cash holding moves into a state with a lower payout boundary.

Similarly, when the firm suddenly transits from the state $s-$ with the financing boundary \underline{w}_{s-} into the other state s with a higher financing boundary ($\underline{w}_s > \underline{w}_{s-}$) and its cash holding lies between the two lower financing boundaries ($\underline{w}_{s-} < w < \underline{w}_s$) it is then optimal for the firm to immediately issue external equity and restore its cash balance to the target level m_s . The following equation describes this rebalancing:

$$p_s(w) = p_s(m_s) - \phi_s - (1 + \gamma_s)(m_s - w), \quad w \leq \underline{w}_s. \quad (21)$$

In the remainder of the paper, we use this model framework to study several scenarios. In Section 4, we consider the case where the firm is attempting to survive a *financial crisis* during with financial markets are temporarily shut down. In Section 5, we consider the situation where the firm expects to transit from the good state, denoted by G , in which external costs of financing are low, to the other state, denoted by B , where the costs of financing are high. And in section 8 we consider the general case where the firm's environment

transits between two *recurrent* states B and G .

4 Fighting for Survival in a Crisis

Our first scenario captures the situation faced by firms in the midst of a financial crisis. Much empirical work has shown, firms in such an environment scramble to survive by cutting back capital expenditures, drawing down lines of credit, and (when possible) engaging in asset sales so as to preserve cash.¹¹ In this section we analyze how firms optimally manage their finances when their priority is to survive in a severe but temporary financial crisis. To make our notation more intuitive, we use state G to refer to the good state, in which financial markets operate normally. We set the fixed cost of equity issuance to 1% of the firm's capital stock in this state ($\phi_G = 1\%$) and the marginal cost of issuance to $\gamma_G = 6\%$. We also set the liquidation value of assets to $l_G = 1.1$. State B is the financial crisis (bad) state, where the market for external financing shuts down. Should the firm run out of cash in this state it would be forced into liquidation. During a financial crisis, few investors have either sufficiently deep pockets or the risk appetite to acquire assets. This leads to fire sale prices of assets and low liquidation values¹² For these reasons, we set $l_B = 0.7$.

The other parameters remain the same in the two states: the riskfree rate is $r = 4.34\%$, the risk-adjusted mean and volatility of the productivity shock are $\hat{\mu} = 21.2\%$ and $\sigma = 20\%$, the rate of depreciation of capital is $\delta = 15\%$, the adjustment cost parameters are $\theta = 6.902$ and $\nu = 12\%$.¹³ Finally, the cash-carrying cost is $\lambda = 1.5\%$. Although in reality these parameter values clearly change with the state of nature, we keep them fixed under this scenario so as to isolate the effects of changes in external financing conditions. All the parameter values are annualized whenever applicable and summarized in Table 1.

To make our point in the simplest possible setting, consider a firm currently in the

¹¹See Campello, Graham, and Harvey (2009), Ivashina and Scharfstein (2009), and Campello, Giambona, Graham, and Harvey (2010).

¹²See Shleifer and Vishny, 1992, Acharya and Viswanathan, 2010, Campello, Graham and Harvey, 2009).

¹³Other than the volatility parameter, we rely on the technology parameters estimated by Eberly, Rebelo, and Vincent (2009).

financial crisis (state B), and that the state G is absorbing, i.e., once the firm reaches state G , it remains there permanently ($\zeta_G = 0$). The firm exits the crisis state with transition probability $\zeta_B \Delta$ over time period Δ , and as a benchmark we set $\zeta_B = 0.9$, which implies that the average duration of a financial crisis is 1.1 years. Under the risk-neutral measure, with a pricing of risk with respect to changes in the state of nature of $\kappa_G = -\kappa_B = \ln(3)$, the corresponding risk-neutral transition intensity is $\hat{\zeta}_B = 0.3$.

The firm's behavior in the absorbing state G is identical to that in the model with constant financing opportunities in BCW. Figure 1 plots the average q and its derivative, as well as the investment-capital ratio $i(w)$ and its derivative in this state. The average q is a natural measure of the value of capital. It is defined as the ratio between the firm's *enterprise value*, $P(K, W, s) - W$, and its capital stock:

$$q_s(w) = \frac{P(K, W, s) - W}{K} = p_s(w) - w. \quad (22)$$

The sensitivity of average q to changes in cash holdings is thus given by

$$q'_s(w) = p'_s(w) - 1. \quad (23)$$

We may interpret $q'_s(w)$ as the (net) marginal value of cash, as it measures how much the firm's *enterprise value* increases with an extra dollar of cash. The firm's investment-capital ratio $i_s(w)$ and investment-cash sensitivity $i'_s(w)$ in each state are given by equations (12) and (13), respectively.

After reaching the absorbing state G , the firm's financing follows a strict *pecking order* with internal funds always tapped before external funds, so that $\underline{w}_G = 0$. The return target for equity issuance, which is also the total amount of equity issuance due to $\underline{w}_G = 0$, is $m_G = 0.17$, and the payout boundary is $\bar{w}_G = 0.49$ (each marked by a vertical line in the graphs). As w rises, the financial constraint is relaxed. As a result, both the average q and investment rise with w , while the net marginal value of cash and the investment-cash sensitivity fall with w . Obviously, the transition intensity ζ_B into the absorbing state has

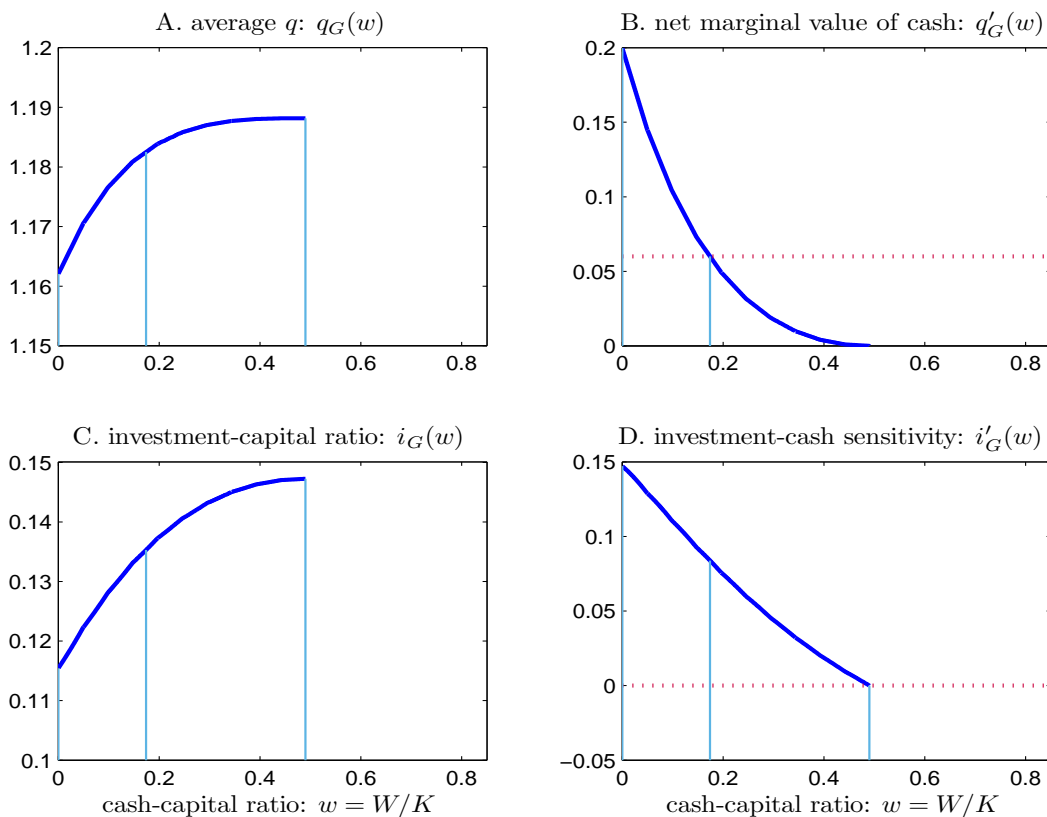


Figure 1: **Firm value and investment in (absorbing) state G .** This figure plots the average q and investment in state G for the case where G is absorbing. Costly external financing is available in state G , but not in B . All parameter values are given in Table 1.

no impact on the results in the absorbing state G .

Next, we turn to the crisis state B , where the firm's overriding concern is *survival* due to the lack of any external financing. The firm also anticipates an improvement in financing opportunities when the state of the economy switches back to normal. Thus, a rise in the probability of leaving the crisis state can have two effects. First, it might encourage the firm to invest with the hope that external financing will become available soon. Second, it raises the continuation value for the firm, which makes the firm place extra weight on survival in order to preserve its going concern value. The tradeoff between these two effects determines how the firm *times payout and investment* in the crisis state.

Figure 2 plots the average q and investment in state B . Panel A plots $q_B(w)$ and gives

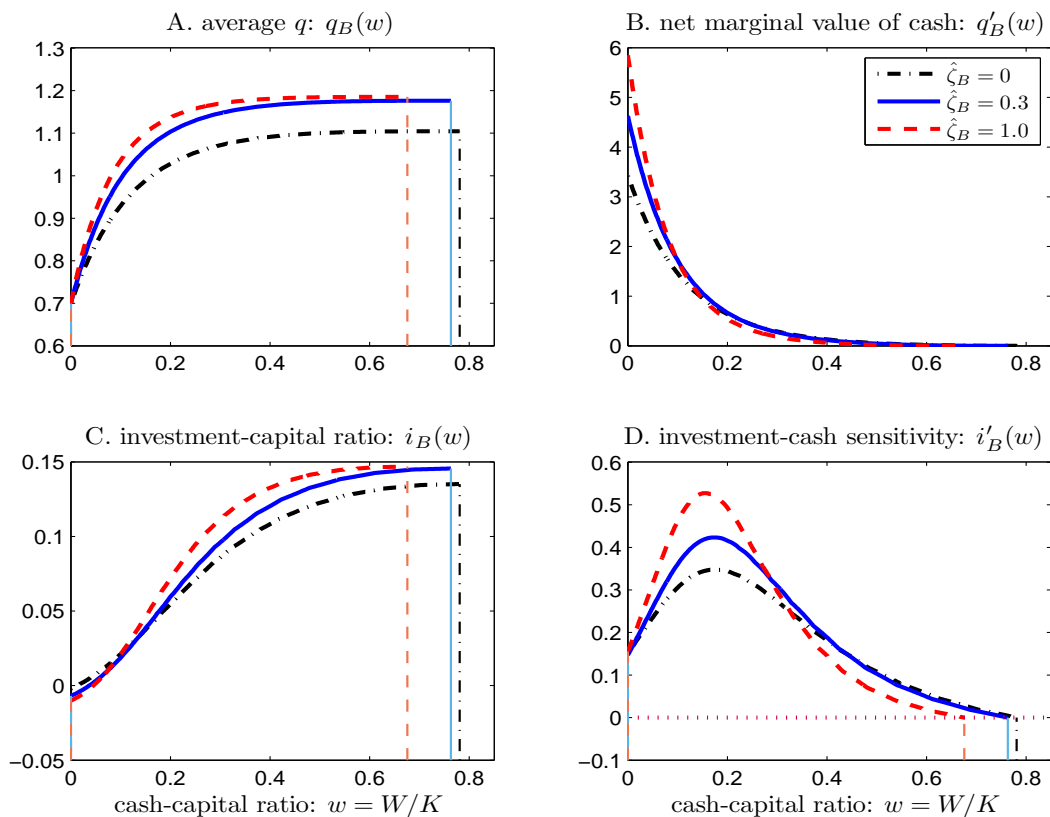


Figure 2: **Firm value and investment in (transitory) state B .** This figure plots the average q and investment in state B for the case where G is absorbing. Costly external financing is available in state G , but not in B . We consider three risk-neutral transition intensities $\hat{\zeta}_B = 0, 0.3, 1.0$. All other parameter values are given in Table 1.

the optimal payout boundary \bar{w}_B in the transitory state B . We consider three levels of risk-neutral transition intensity, $\hat{\zeta}_B = 0, 0.3, 1$, which corresponds to $\zeta_B = 0, 0.9, 3$ under the physical measure. Regardless of the transition intensity, the average q always starts at $l_B = 0.7$ due to liquidation at $w = 0$. When the probability of exiting a crisis increases, firm value rises, and the firm responds by reducing its cash holding. The payout boundary \bar{w}_B falls from 0.78 to 0.76 and then to 0.68 as $\hat{\zeta}_B$ rises from 0 to 0.3 and then to 1.0.

It is worth noting that the payout boundary in state B is not always monotonic in $\hat{\zeta}_B$. For very high and very low transition intensities the firm pays out sooner than for intermediate intensities. The reason is that when the firm is stuck in the crisis state for a long time the

value of its investment opportunities is so low that it is best to payout cash to shareholders. When the probability of exiting the crisis is very high then the prospect of raising cheap equity in the future also encourages the firm to pay out more dividends in the crisis state. It is for intermediate probabilities, when the value of the firm's investment opportunities is relatively high, but the risk of staying in a prolonged crisis is also high, that the firm is most conservative in its payout policy.

However, even when the crisis is expected to end quickly (e.g., $\hat{\zeta}_B = 1$ corresponds to $\zeta_B = 3$, which implies the average duration of state B is only 0.33 years), the payout boundary is still significantly higher than in the good state ($\bar{w}_G = 0.49$), suggesting that the firm has a strong desire to hold more cash in the crisis state. The graph also shows that moving from state B to G can result in a big jump in firm value when the cash holding is low, but the effect is much smaller when the cash holding is high. This difference reflects the fact that the firm uses precautionary savings to cushion the impact of severe financial constraints. One implication of this finding is that we should not expect to see sharp increases in stock valuations for cash rich firms as the economy exits the crisis state.

Panel B plots the net marginal value of cash $q'_B(w)$ in state B . As w approaches 0, the marginal value of cash rises significantly because an extra dollar of cash can reduce the chance of costly liquidation. While the net marginal value of cash in state G reaches at most \$0.2 as $w \rightarrow 0$, it can be as high as \$6 in state B . Again, this is due to the fact that the firm has access to external financing in state G but not in state B .

Interestingly, when cash holdings w are relatively high the marginal value of cash in state B decreases with the transition intensity $\hat{\zeta}_B$, while it increases with $\hat{\zeta}_B$ when w is low. This result might appear counter-intuitive, as a higher probability of ending the crisis ought to help relax the financial constraint the firm is facing. Intuitively, the severity of financial constraints depends on the probability of the firm running out of cash before the crisis ends. When current cash holding is high, a higher $\hat{\zeta}_B$ makes liquidation less likely, hence reducing the importance of hoarding cash today. However, when the firm is facing an immediate liquidation threat, yet the chance of the crisis ending in the near future is high,

the “breathing room” provided by an extra dollar of cash can be especially valuable, which explains why the marginal value of cash rises with $\hat{\zeta}_B$. Notice that for $w > 0$, the marginal value of cash should eventually decrease in $\hat{\zeta}_B$ as $\hat{\zeta}_B$ becomes large, since the high intensity eventually makes liquidation concerns irrelevant.¹⁴

The behavior of the marginal value of cash is key to understanding the firm’s investment policy. As Panel C shows, the investment-capital ratio $i_B(w)$ in state B is increasing in w . This result is driven by the rise in firm value and the fall in marginal value of cash with w . With sufficiently high w , investment increases with $\hat{\zeta}_B$. But the opposite is true when w is low. Underinvestment is a form of risk management for a financially constrained firm. When the firm does not face an immediate threat of liquidation, a higher transition intensity $\hat{\zeta}_B$ further reduces the need to save cash and hence makes the firm more willing to invest. However, if the cash holding is already low, a higher $\hat{\zeta}_B$ can induce the firm to underinvest more in order to avoid running out of cash before the end of the crisis. The different investment policy at the lower and higher ends of w highlights the importance of a dynamic risk management perspective.

Panel D of Figure 2 shows that the investment-cash sensitivity $i'_B(w)$ is positive but non-monotonic in w . Kaplan and Zingales (1997) show that investment increases with net worth ($i'(w) > 0$) but cannot sign $i''(w)$ in their static setting. In the scenario we illustrate here, the sensitivity $i'(w)$ is positive, and indeed can be either increasing or decreasing in w .

In summary, when current external financing is impossible but may be available in the future, the potential change of financing terms in the future affects the firm’s payout and investment policies. From the comparative statics for $\hat{\zeta}_B$, we can conjecture the implications of a time-varying transition intensity in a dynamic setting. When $\hat{\zeta}_B$ rises, which can be either because the expected duration of the crisis is getting shorter ($\hat{\zeta}_B$ falls), or because investors are less concerned with the crisis state (the risk premium for financing shocks falls), firm value will rise, firms will tend to hold less cash, and investment may be falling for firms with low cash holdings (despite the fact that expected future financing costs are falling) but

¹⁴The exception is at the limit as w approaches 0, where one can prove that the marginal value of cash will be monotonically increasing in $\hat{\zeta}_B$.

rising for firms with high cash holdings.

5 Market Timing: Building a War-chest in Good Times

In this section, we consider a setting where the firm is currently in state G . However, the economy may switch out of state G to enter the crisis state B with probability $\zeta_G \Delta$ over the time interval Δ . Moreover, in state B the firm cannot access external financing and can only survive on internal funds. Thus, under the scenario considered in this section the firm has an external financing window only in state G , and this window has limited duration. Unlike in the previous section, we show that the option to time the market has significant value.

This predictable worsening of financing conditions generates a positive timing-option value for the firm. By tapping external equity markets while there is still time, the firm can build a *cash war chest* for the future. By deferring external financing, it would save on the time value of money for financing costs and also on subsequent cash carry costs. However, doing so would then take a risk of being shut out of capital markets forever before it had time to accumulate cash. Facing this tradeoff, the firm chooses its external equity issuance policy together with its investment and payout policies to maximize its value.

The firm's behavior in the absorbing state B is essentially the same as in BCW. Figure 3 plots the average q and $i(w)$ in the absorbing state B . If the firm runs out of cash in state B , the inability to raise external funds results in immediate liquidation. Average q thus is equal to the liquidation value $l_B = 0.7$ at $w = 0$. Also, average q is concave in w (as in BCW). The net marginal value of cash $q'_B(w)$ can be as high as 3.5 when the firm is close to running out of cash, but it decreases to 0 monotonically as we w increases from 0 to the endogenous payout boundary \bar{w}_B . As in BCW, investment is increasing but is not necessarily concave in cash: from Panels C and D one can see that $i'(w)$ is positive but not monotonic.

Next we turn to the transitory state G , Figure 4 plots firm value, investment, and their sensitivities in state G for three levels of risk-neutral transition intensity $\hat{\zeta}_G = 0, 0.3, 1.0$ from state G to B , which corresponds to $\zeta_G = 0, 0.1, 1/3$ under the physical measure.

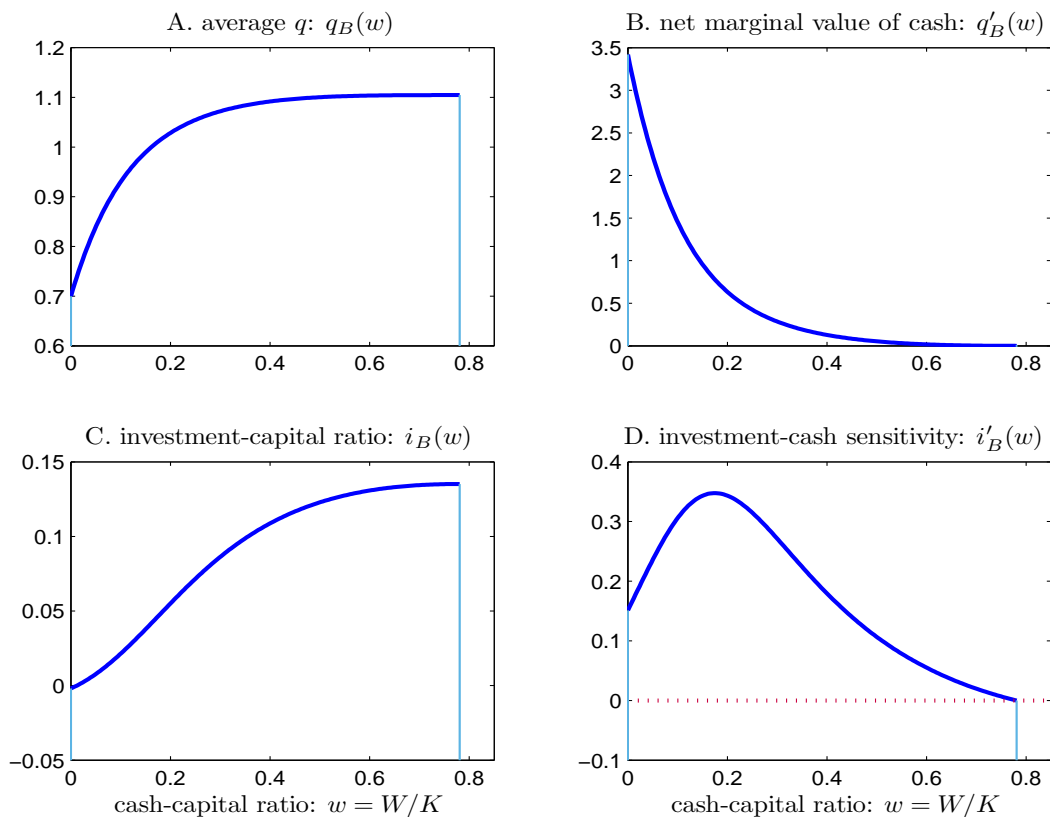


Figure 3: **Firm value and investment in (absorbing) state B .** This figure plots the average q and investment in state B for the case where B is absorbing. Costly external financing is not available in state B . All parameter values are given in Table 1.

Panel A plots average q . Intuitively, the higher is the transition intensity from G to B (the higher $\hat{\zeta}_G$) the lower is firm value for the same cash-capital ratio w . Importantly, when $\hat{\zeta}_G$ is sufficiently high firm value is no longer globally concave in w . Since financial constraints typically induce the firm to hoard cash for precautionary reasons, firm value is increasing and concave in financial slack in almost all models featuring financial constraints. In our scenario, the precautionary motive for hoarding cash is still present. Yet, stochastic financing conditions also introduce a motive to time equity markets, which potentially results in a locally convex firm value.

From Panel B, it is easy to see that firm value is not globally concave in w . For sufficiently high w ($w \geq 0.17$ with $\hat{\zeta}_G = 0.3$ and $w \geq 0.26$ with $\hat{\zeta}_G = 1$) $q_G(w)$ is concave. When the

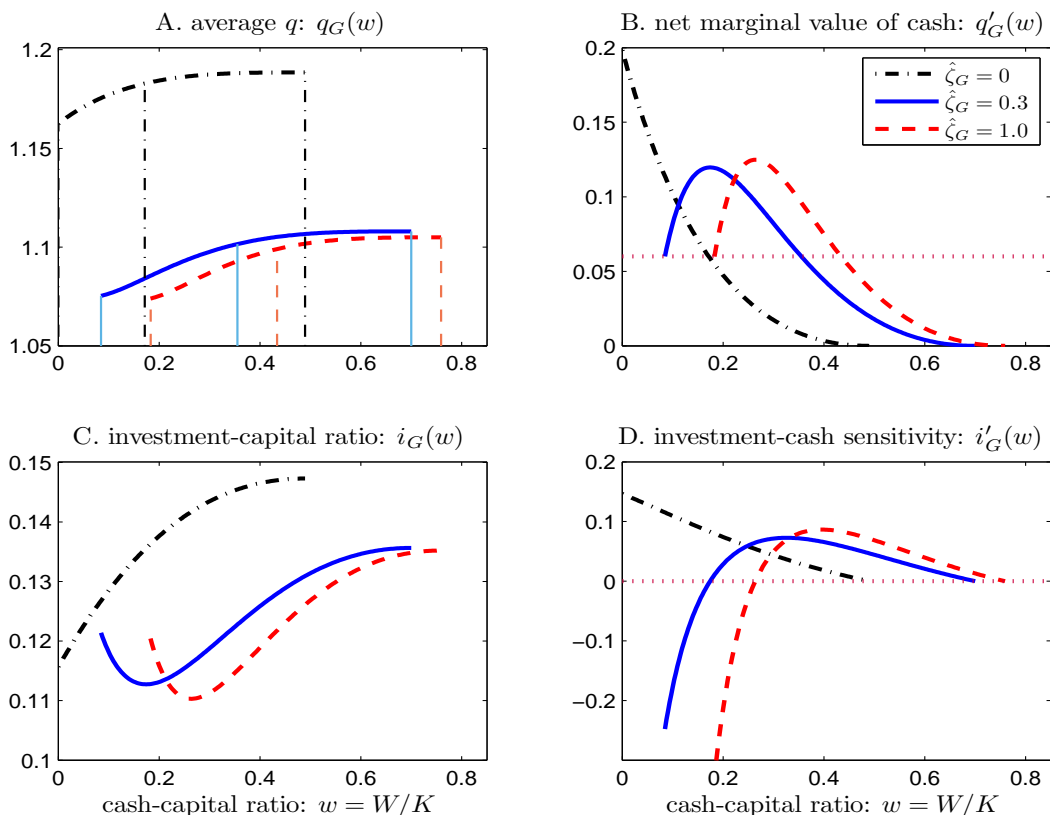


Figure 4: **Firm value and investment in (transitory) state G .** This figure plots the average q and investment in state G for the case where G is transitory. Costly external financing is available in state G , but not in B . We consider three transition probabilities $\hat{\zeta}_G = 0, 0.3, 1.0$. All other parameter values are given in Table 1.

firm has sufficient cash, the firm's equity issuance need is then quite distant so that the financing timing option is *out-of-the-money*. Recall that the sign of $i'(w)$ is determined by $p_G''(w)$ (see equation 13). Hence, the concavity of $p_G(w)$ in the cash rich region also implies that investment responds positively to increases in cash in that region, which is confirmed in Panels C and D of Figure 4. To sum up, with sufficient financial slack, the firm behaves effectively in the same way as in standard models with financial constraints (e.g. BCW).

In contrast, when w is low (e.g. $w \leq 0.17$ with $\hat{\zeta}_G = 0.3$ and $w \leq 0.26$ with $\hat{\zeta}_G = 1$) the firm is more concerned about the risk of being shut out of capital markets when the state switches to B . A firm with low cash holdings may want to issue equity while it can, even

before running out of cash. In addition, such a firm may choose to accelerate its *cash burn rate* by increasing its investments, to bring forward the time when it raises new funds through equity issuance. Also, due to the fixed costs of issuing equity, the firm will engage in a lumpy equity issue when it chooses to tap equity markets. This means that post issuance the firm will have high cash holdings. Thus when w is low, the expectation of high post issuance cash reserves coupled with the inclination to time favorable equity markets dominates the firm's precautionary motive, resulting in a locally convex-shaped firm value in w .

How does the transition intensity out of state G affect firms' market timing motive? Consider first the limiting case when state G is absorbing ($\hat{\zeta}_G = 0$). In this case, the firm taps equity markets only when it runs out of cash ($\underline{w}_G = 0$), and to economize the fixed cost of issuance, the firm issues a lumpy amount $m_G = 0.17$. Firm value $q_G(w)$ is then globally concave in w and $i_G(w)$ increases with w everywhere. Note in particular that the fixed issuance cost by itself is not sufficient to generate market timing behavior. The transitory nature of favorable market conditions is necessary to induce the firm to time the market. As the transition intensity $\hat{\zeta}_G$ rises above 0 the equity issuance boundary \underline{w}_G may possibly move above 0. In these situations, the optimality condition for the issuance boundary requires that the net marginal value of cash at the issuance boundary be equal to the proportional financing cost $\gamma = 6\%$. As one would expect, the return cash-capital ratio, m_G , is also increasing in the transition intensity (as can be seen in Panel A), since a higher likelihood of an impending financial crisis raises the firm's precautionary demand for cash. The firm also chooses to preserve more cash in response to an increase in $\hat{\zeta}_G$ by postponing payouts to shareholders. This can be seen from the shift to the right for the optimal payout boundary \bar{w}^G as $\hat{\zeta}_G$ rises. In sum, Panel A shows that through a combination of market timing and reduced payout, the firm optimally responds to a greater crisis risk by holding more cash on average.

Besides the finite duration of the option to time the equity market, the fixed issuance cost is also necessary to obtain local convexity of the value function. In Figure 5, we examine average q and investment in the transitory state G for three levels of fixed cost of equity

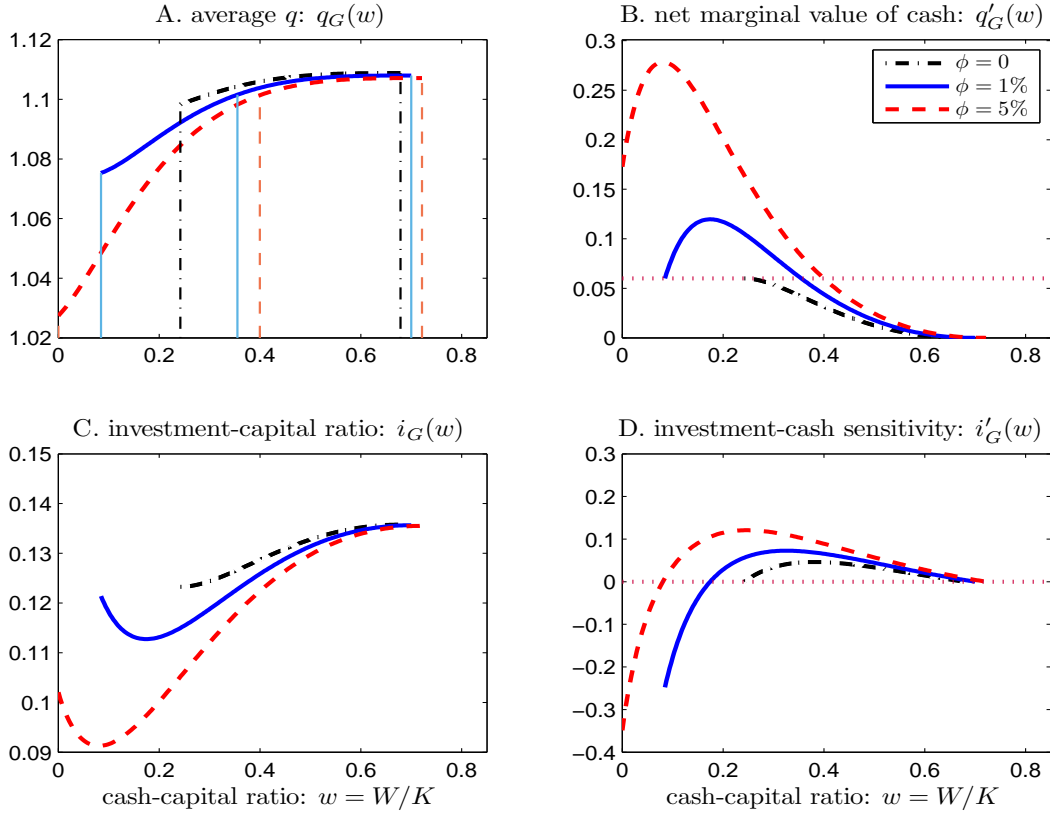


Figure 5: **The effects of financing costs on firm value and investment in (transitory) state G .** This figure plots firm value and investment in state G for the case where G is transitory. We consider three levels of fixed costs of equity financing in state G : $\phi_G = 0, 1\%, 5\%$. The transition intensity is $\hat{\zeta}_G = 0.3$. All other parameter values are given in Table 1.

financing $\phi = 0, 1\%, 5\%$ (with transition intensity $\hat{\zeta}_G = 0.3$). Note first that the lower the fixed cost parameter ϕ is the earlier the firm issues equity in state G . Intuitively, the firm exercises the financing option earlier if the cost of doing so is lower. In Panel A, as ϕ_G drops from 5% to 1% and then to 0, the financing boundary \underline{w} rises from 0 to 0.08 and then to 0.24. Second, without the fixed cost ($\phi_G = 0$), the firm issues just enough equity to stay away from its optimally chosen financing boundary \underline{w} , as the net marginal value of cash cannot be higher than the marginal cost of financing γ . In this extreme case, the marginal value of cash $q'_G(w)$ is monotonically decreasing in w as can be seen from Panel B; hence, firm value is globally concave in w even under market timing. Thus, stochastic financing

costs and fixed costs are both necessary to generate convexity.¹⁵

When the fixed cost of issuing equity is positive but not very high ($\phi_G = 1\%$ or 5%) the marginal value of cash is no longer monotonic in w . Moreover, higher fixed costs lead firms to choose larger issuance sizes ($m_G - \underline{w}_G$). Notice also that $\underline{w}_G = 0$ when $\phi_G = 5\%$. This result shows that market timing does not necessarily lead to a violation of the pecking order between internal cash and external equity financing, and importantly that $\underline{w}_G > 0$ is not necessary for the convexity of the value function. Finally, when the fixed cost of issuing equity is very high (not shown in the graph), the market timing effect is so weak that the precautionary motive dominates again, so that the net marginal value of cash is monotonically decreasing in w .

Having determined why the value function may be locally convex, we now explore the implications of convexity for investment. Recall from equation (13) that the sign of the investment-cash sensitivity $i'_s(w)$ depends on $p''_s(w)$. Thus, in the region where $p_G(w)$ is convex, *investment is decreasing in cash holdings w* . This finding is in sharp contrast to the standard result in the investment with financial constraint literature, where investment is always positively related with w . Indeed, in all existing models with financial constraints investment increases with financial slack. Existing models only differ in their results concerning investment-cash sensitivity (the sign of the second derivative $i''(w)$).¹⁶

The economic reason for why investment may be locally decreasing in financial slack is related to market timing. When the firm's cash holding is low, it wants to take advantage of the favorable financing condition in state G before it disappears. As a result, rather than cutting investment further to avoid further reducing the cash holding, the firm actually wants to accelerate investment in order to reach the equity issuance boundary sooner. Also anticipating equity issuance, it is less worthwhile for the firm to significantly distort investment. The optionality of issuing equity generates convexity. Simply put, the firm is

¹⁵More generally, the value of the market timing option depends on the difference in financing costs between the two states: either lowering the financing costs in state G or raising the financing costs in state B (or making liquidation more costly in state B) will lead the firm to issue equity early.

¹⁶See Kaplan and Zingales' (KZ) discussion of Fazzari, Hubbard, and Petersen (1988). See also Stein (2003) for a survey on the issue of investment/cash sensitivity.

more willing to invest when higher investment reduces the chance of missing out on favorable equity financing conditions, and when it expects its cash holding to substantially increase post equity issue. This behavior is shown in Panels C and D of Figure 4 and 5. Our model is thus able to account for the behavior that the threat of high financing costs in the future can cause changes in investment and cash holding to be negatively correlated.

There may be other ways of generating a negative correlation between changes in investment and cash holding. First, when the firm moves from state G to B , this not only results in a drop in investment, especially when w is low (comparing Panel C in Figure 3 and 4), but also in an increase in the payout boundary, which may explain why firms during the recent financial crisis have increased their cash reserves and cut back on capital expenditures, as Acharya, Almeida, and Campello (2010) have documented. Second, in a model with persistent productivity shocks (as in Riddick and Whited (2009)), when expected future productivity falls, the firm will cut investment and the cash saved could also result in a rise in its cash holding.¹⁷

Is it possible to distinguish empirically between these two mechanisms? In the case of a negative productivity shock the firm has no incentive to significantly raise its payout boundary, as lower productivity lowers the costs of underinvestment, hence reducing the precautionary motive for holding cash. This prediction is opposite to the prediction related to a negative financing shock. Thus, following negative technology shocks we will not see firms aggressively increasing cash reserves. In fact, firms that already have high cash holdings will likely pay out cash after a negative productivity shock, but hold on to even more cash after a negative financing shock.

Another empirical prediction which differentiates our model from other market timing models concerns the link between equity issuance and corporate investment. Our model predicts that underinvestment is substantially mitigated when the firm is close to the equity financing boundary. Moreover, the positive correlation between investment and equity issuance in our model is not driven by better investment opportunities (as the real side of

¹⁷This mechanism is captured in our model with the two states corresponding to two different values for the return on capital μ_g .

the economy is held constant across the two states) it is driven solely by the market timing and precautionary demand for cash.

6 Financial Constraints and the Risk Premium

In this section, we explore how financial constraints and time-varying external equity issuance costs affect the firm's cost of capital. A heuristic derivation of the firm's (risk-adjusted) expected return involves a comparison of the HJB equations under the physical and risk-neutral measures \mathbb{P} and \mathbb{Q} . Let the firm's conditional risk premium in state s be $\mu_s^R(w)$. We may write the HJB equation under the physical measure as follows

$$\begin{aligned} (r_s + \mu_s^R(w)) p_s(w) = \max_{i_s} [(r_s - \lambda_s) w + \mu_s - i_s - g_s(i_s)] p'_s(w) + \frac{\sigma_s^2}{2} p''_s(w) \\ + (i_s - \delta) (p_s(w) - w p'_s(w)) + \zeta_s (p_{s^-}(w) - p_s(w)), \end{aligned} \quad (24)$$

where μ_s and ζ_s denote the expected excess return on capital and the transition intensity from state s to s^- under the physical probability measure, respectively.

By matching terms in the HJB equations (11) and (24), one then obtains the following expression for the *conditional risk premium*:

$$\mu_s^R(w) = \eta_s \rho_s \sigma_s \frac{p'_s(w)}{p_s(w)} - (e^{\kappa_s} - 1) \zeta_s \frac{(p_{s^-}(w) - p_s(w))}{p_s(w)}, \quad (25)$$

where ρ_s is the conditional correlation between the firm's productivity shock dA and the stochastic discount factor in state s .¹⁸ The first term in (25) is the technology risk premium, which is the product of the firm's exposure to systematic Brownian risk $\rho_s \sigma_s p'_s(w)/p_s(w)$ and the price of Brownian risk η_s . It is positive for firms whose values are positively correlated with aggregate technology shocks. The ratio $p'_s(w)/p_s(w)$ measures the percentage change of firm value with respect to a unit change in w . The second term is the financing risk

¹⁸This expression can also be obtained via the standard covariance between return and stochastic discount factor derivation (see e.g., Duffie (2001)).

premium, which compensates risk-averse investors for the exposure to the firm's risk with respect to time-varying equity issuance costs. Since the stochastic discount factor (marginal utility) jumps up when financing conditions deteriorate we naturally have $\kappa_G = -\kappa_B > 0$ in our two state model: risk-averse investors demand this extra premium for firms whose values drop during times when external financing conditions worsen ($p_G(w) > p_B(w)$).¹⁹

Note that in the *first-best* setting where there are no equity issuance costs, the firm's expected risk premium is constant and can be recovered from (25) by setting η , ρ , and σ to be constants and dropping the second term. We then obtain the standard CAPM formula:

$$\mu^{FB} = \eta\rho\sigma\frac{1}{q^{FB}} = \beta^{FB}(r_m - r), \quad (26)$$

where $\beta^{FB} = \rho\sigma/(\sigma_m q^{FB})$, $(r_m - r)$ is the excess market portfolio return, and σ_m is the market portfolio volatility.

The comparison between $\mu^R(w)$ and μ^{FB} highlights the impact of external financing frictions on the firm's cost of capital:

Constant equity issuance costs: When financing opportunities are constant over time, financial constraints only affect the cost of capital by amplifying (or dampening) a firm's exposure to technology shocks. This effect is captured by the technology (diffusion) risk premium in (25). As the cash-capital ratio w increases, the firm tends to become less risky for two reasons. First, if a greater fraction of its assets is cash, the firm beta is automatically lower due to a simple portfolio composition effect. As a financially constrained firm hoards more cash to reduce its dependence on costly external financing, the firm beta becomes a weighted average of its asset beta and the beta of cash, which is equal to zero. In particular, with a large enough buffer stock of cash relative to its assets, this firm may be even safer than a firm facing no external financing costs and therefore holding no cash. Second, an increase in w effectively relaxes the firm's financing constraint and therefore reduces the sensitivity of firm value

¹⁹Livdan, Sapriza, and Zhang (2009) also study the effect of financing constraints on stock returns. Their model, however, does not allow for stochastic financing conditions or cash accumulation.

to cash flow, which also tends to reduce the risk of holding the firm.

Time-varying equity issuance costs: Time-varying equity issuance costs affect the cost of capital for a financially constrained firm in two ways. First, the firm’s exposure to technology shocks changes as financing conditions change, as the marginal value of cash $p'_s(w)$ and firm value $p_s(w)$ both depend on the state s . Second, when external financing shocks are priced, investors demand an extra premium for investing in firms that do poorly when financing conditions worsen. This effect is captured by the second term in (25). Note that $(p_{s-}(w) - p_s(w)) / p_s(w)$ gives the percentage change of firm value if financing conditions change, and this term measures the sensitivity of firm value with respect to changes in w . Intuitively, the financing risk premium is larger the bigger the relative change in firm value due to a change in external financing conditions.

Figure 6 plots the conditional risk premium for a firm as a function of w . Recall that state G (with low financing costs) is transient and state B (with high financing costs) is absorbing. We set the price of Brownian risk to $\eta_G = \eta_B = 0.4$, and the correlation between the aggregate and firm level Brownian shocks to $\rho_G = \rho_B = 0.6$. The remaining parameters are the same as in the benchmark case and are reported in Table 1. The risk premium for an unconstrained firm in the first-best setting is then $\mu^{FB} = 3.2\%$.

In Panel A, the *total risk premium* in state B is shown to be decreasing in the firm’s cash holding. When w is close to 0, the annualized conditional risk premium can exceed 30%, but it falls rapidly as w rises. This result mirrors the rapid decline in the marginal value of cash (see Figure 3, Panel B): thus, high marginal value of cash in the low w region can dramatically amplify the firm’s sensitivity to technology shocks relative to the unconstrained case. The risk premium can be as low as 2% for a firm near the payout boundary—even lower than the total risk premium for a financially unconstrained firm (3.2%). This is due to the asset composition effect discussed earlier. As the firm approaches the payout boundary, the marginal value of cash $p'_B(w)$ approaches 1. By definition, $p_B(w) = q_B(w) + w$. While the average q for the constrained firm will always be below the q under the first best, q^{FB} , the sum of average q and w can exceed q^{FB} , which causes $\mu_B^R(w)$ to fall below μ^{FB} .

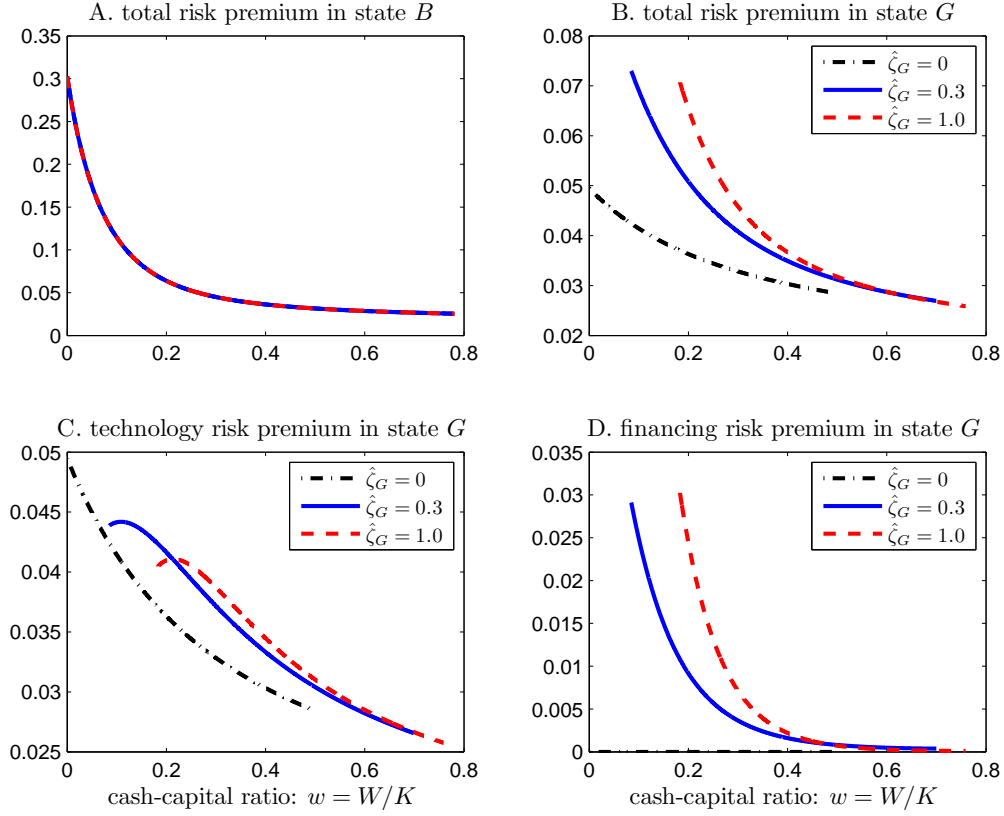


Figure 6: **The effects of stochastic financing conditions on the cost of capital.** This figure plots firm risk premium in state G and B for the case where G is transitory. We consider three levels of transition intensity from G to B : $\hat{\zeta}_G = 0, 0.3, 1.0$.

In Panel B, the total risk premium in state G also decreases when the firm's cash holding rises. Compared to state B , the level of risk premium is lower, especially when the firm has low cash holdings. Moreover, a higher probability of switching into state B raises the risk premium. This effect is weaker for a firm with a higher cash-capital ratio as the composition effect then becomes stronger.

When we decompose the total risk premium in state G into the technology and financing components we observe the following. First, Panel C plots the technology risk premium (the first term in (25)). As can be seen from this panel, when the external financing conditions do not change ($\hat{\zeta}_G = 0$) the technology risk premium is monotonically decreasing in w . However, under time-varying external financing costs, the risk with respect to higher future financing

costs generates market timing behavior and non-monotonicity in the marginal value of cash (Figure 4, Panel B), which in turn may involve a technology risk premium that is locally increasing in w for low levels of w . As the non-monotonicity in the marginal value of cash is partially offset by the asset composition effect, the non-monotonicity in the technology risk premium is relatively weak. Similarly, holding w fixed at a low level, market timing can lower $p'_G(w)$ as the transition intensity $\hat{\zeta}_G$ increases. This explains why the technology risk premium may be decreasing in the transition intensity for low w .

Second, Panel D plots the financing risk premium. The size of this premium depends on the relative change in firm value when external financing conditions change. It is increasing in the transition intensity $\hat{\zeta}_G$, but decreasing in w . Intuitively, when cash holdings are low, a sudden worsening in external financing conditions is particularly costly, but when cash holdings are high, the firm is able to avoid liquidation by cutting investment, engaging in asset sales, and deferring payout, all of which mitigate impact of the financing shock.

Our model has several implications for expected returns of financially constrained firms. Controlling for technology parameters and financing costs, the model predicts an inverse relation between returns and corporate cash holdings, which has been documented by Dittmar and Mahrt-Smith (2007) among others. Our analysis points out that this negative relation may not be due to agency problems, as they emphasize, but may be driven by relaxed financing constraints and a changing asset composition of the firm.²⁰

A related prediction is that firms that are more financially constrained are not necessarily more risky. The risk premium for a relatively more constrained firm can be lower than that for a less constrained firm if the more constrained firm also holds more cash. This observation may shed light on the recent studies by Ang, Hodrick, Xing, and Zhang (2006, 2009) documenting that stocks with high idiosyncratic volatility have low average returns. In

²⁰When heterogeneity in technology and financing costs is difficult to measure, it is important to take into account the endogeneity of cash holdings when comparing firms with different cash holdings empirically. A firm with higher external financing costs will tend to hold more cash, however its risk premium may still be higher than for a firm with lower financing costs and consequently lower cash holdings. Thus, a positive relation between returns and corporate cash holdings across firms may still be consistent with our model (see Palazzo (2008) for a related model and supporting empirical evidence).

our model, firms that face higher idiosyncratic risk will optimally hold more cash on average, which could explain their lower risk premium.

Finally, with time-varying financing conditions, our model can be seen as a conditional two-factor model to explain the cross section of returns (we provide details of the derivation in the Appendix). A firm's risk premium is then determined by its technology beta and its *financing beta*. Other things equal, a firm whose financing costs move closely with aggregate financing conditions will have a larger financing beta and earn higher returns than one with financing costs independent of aggregate conditions. Empirically, this two-factor model can be implemented using the standard market beta plus a beta with respect to a portfolio that is sensitive to financing shocks (e.g. a banking portfolio). This model, in particular, shows how a firm's conditional beta depends on the firm's cash holdings.

7 Market Timing and Dynamic Hedging

We have thus far restricted the firm's financing choices to only internal funds and external equity financing. In this section, we extend the model to allow the firm to engage in dynamic hedging via derivatives such as market-index futures. How does market timing behavior interact with dynamic hedging? And, how does the firm's dynamic hedging strategy affect its market timing behavior? These are the questions we address in this section. We denote by F the index futures price for a market portfolio that is already completely hedged against financing shocks. Under the risk-neutral probability measure, the future prices F then evolves according to:

$$dF_t = \sigma_m F_t d\hat{Z}_t^M, \quad (27)$$

where σ_m is the volatility of the market index portfolio, and $\{\hat{Z}_t^M : t \geq 0\}$ is a standard Brownian motion that is correlated with the firm's productivity shock $\{Z_t^A : t \geq 0\}$ with a constant correlation coefficient ρ .²¹

²¹Note that the futures price F follows a martingale after risk adjustment. The interesting case to consider is when the index futures is imperfectly correlated with the firm's productivity shock.

We denote by ψ_t the fraction of the firm's total cash W_t that it invests in the futures contract. Futures contracts require that investors hold cash in a *margin account*. Thus, let $\alpha_t \in [0, 1]$ denote the fraction of the firm's total cash W_t held in the margin account. Cash held in this margin account incurs a flow unit cost $\epsilon \geq 0$. Futures market regulations typically require that an investor's futures position (in absolute value) cannot exceed a multiple π of the amount of cash $\alpha_t W_t$ held in the margin account. We let this multiple be state dependent and denote it by $\pi(s_t)$. The margin requirement in state s then imposes the following limit on the firm's futures position: $|\psi_t| \leq \pi(s_t)\alpha_t$. As the firm can costlessly reallocate cash between the margin account and its regular interest-bearing account, it optimally holds the minimum amount of cash necessary in the margin account when $\epsilon > 0$. Without much loss of generality we shall ignore this haircut on the margin account and assume that $\epsilon = 0$. Under this assumption, we do not need to keep track of cash allocations in the margin account and outside the account. We can then simply set $\alpha_t = 1$. Since the derivation of firm value and optimal hedging policy follows closely the analysis in BCW we do not develop it in the text below and provide a more detailed derivation in the Appendix C, where we establish that:

1. in the absorbing state B , the optimal futures position is given by

$$\psi_B^*(w) = \max \left\{ \frac{-\rho\sigma_B}{w\sigma_m}, -\pi_B \right\}.$$

2. in the transitory state G , the optimal futures position is given by

$$\psi_G^*(w) = \begin{cases} \max \{-\rho\sigma_G\sigma_m^{-1}/w, -\pi_G\}, & \text{for } w \geq \hat{w}_G, \\ \pi_G, & \text{for } \underline{w}_G \leq w \leq \hat{w}_G. \end{cases}$$

We choose the correlation between index futures and the firm's productivity shock to be $\rho = 0.6$ and a market return volatility of $\sigma_m = 20\%$. The margin requirements in states G and B are set at $\pi_G = 5$ and $\pi_B = 2$, respectively. All other parameter values are the same as in the previous sections.

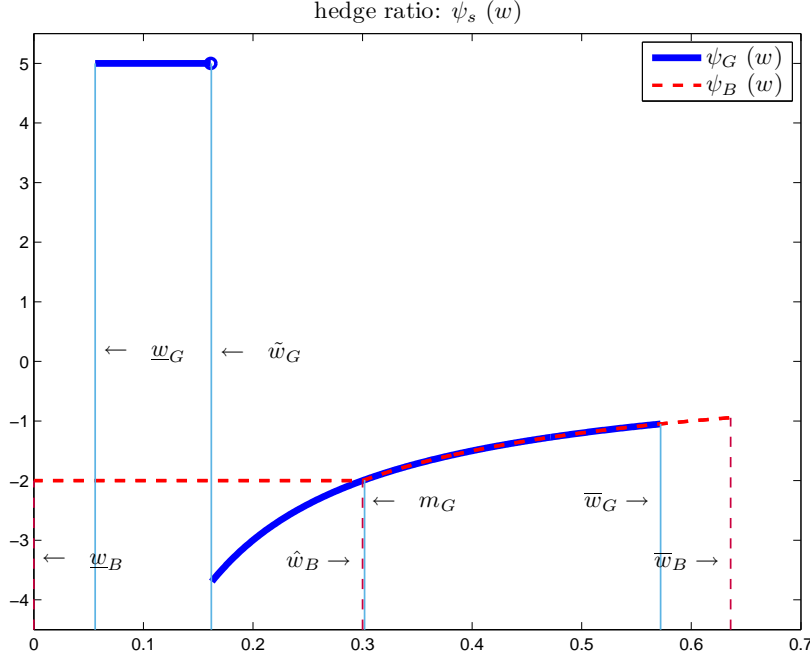


Figure 7: **Optimal hedge ratios $\psi^*(w)$ in states G and B when state B is absorbing.** The parameter values are: market volatility $\sigma_m = 20\%$, correlation coefficient $\rho = 0.6$, margin requirements $\pi_G = 5$ and $\pi_B = 2$. All other parameter values are given in Table 1.

Optimal hedge ratios $\psi_s^*(w)$. Figure 7 plots the optimal hedge ratios in both states: $\psi_G^*(w)$ and $\psi_B^*(w)$. First, we note that for sufficiently high w , the firm hedges in the same way in both states. Hedging is then unconstrained by the firm's cash holdings and costless, so that the firm chooses its hedge ratio to be equal to $-\rho\sigma\sigma_m^{-1}/w$ so as to eliminate its exposure to *systematic volatility* of the productivity shock. This explains the concave and overlapping parts of the hedging policies in Figure 7.

Second, for low w hedging strategies differ in the two states as follows: in state B the hedge ratio hits the constraint $\psi_B^*(w) = -\pi_B = -2$ for $w \leq 0.3$. In state G on the other hand, firm value turns from concave to convex (due to market timing) when w is less than $\tilde{w}_G = 0.16$ (where $p''(\tilde{w}_G) = 0$). For $w \in (\underline{w}_G, \tilde{w}_G)$ firm value is convex in w so that the firm does the opposite of hedging and engages in maximally allowed *risk taking* by setting $\psi_G^*(w) = \pi_G = 5$ for $w \in (0.06, 0.16)$.

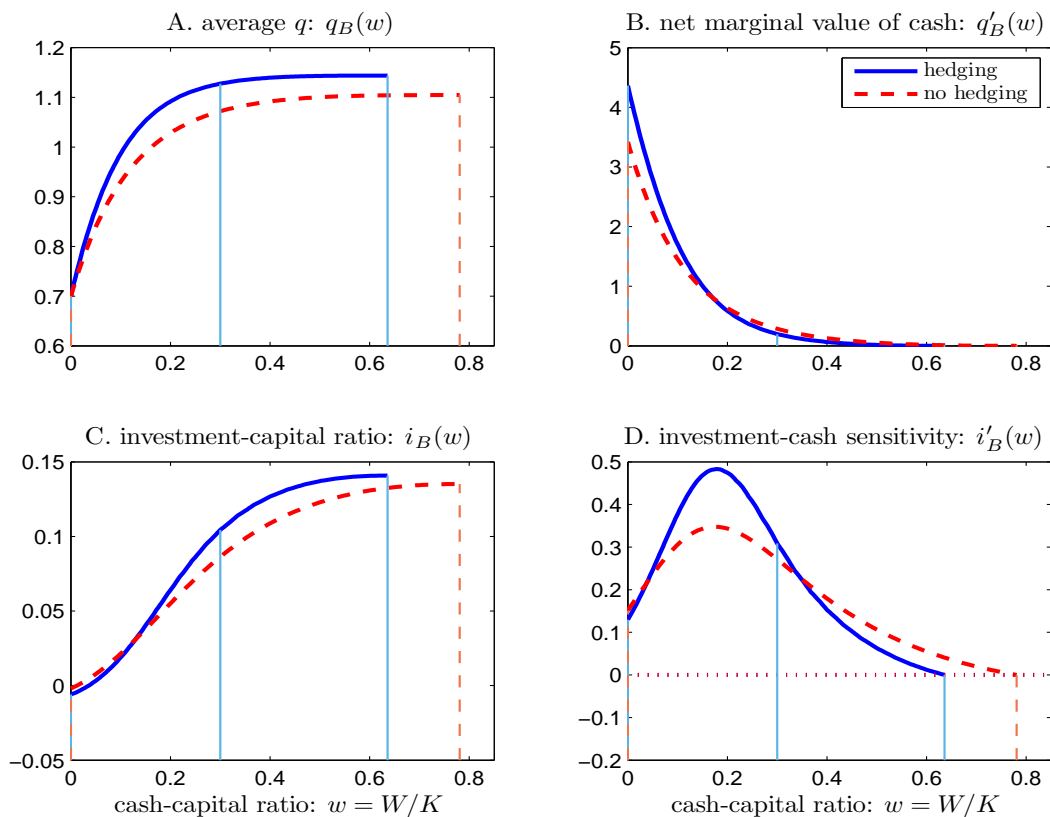


Figure 8: **Firm value and investment in the (absorbing) state B .** This figure plots the average q and investment in state B with and without hedging opportunities. External financing is available in state G , but not in B . For the hedging case, we set market volatility $\sigma_m = 20\%$, correlation coefficient $\rho = 0.6$, and margin requirements: $\pi_G = 5$ and $\pi_B = 2$. All other parameter values are given in Table 1.

Hedging and investment in the absorbing state B . Figure 8 plots firm value $q(w)$ and the investment-capital ratio $i(w)$ as functions of w in the absorbing state B . We compare the solutions with and without hedging. As in BCW, firm value $q(w)$ is higher with hedging than without (Panel A). Also, when w is sufficiently high the net marginal value of cash $q'_B(w)$ is higher for firms that do not hedge than for those that do. This is because cash plays a more important role in risk management when there are no other hedging tools available. However, when cash is low, the marginal value of cash is higher when the firm hedges than when it does not. This is due to the fact that the firm is more valuable with hedging opportunities in the future than without. Hence, the marginal value of cash is greater for firms with better

future prospects.

Similarly, investment-capital ratios on average are higher for firms that also hedge with futures and for firms with sufficiently high cash. However, when cash is low disinvestment becomes a risk management tool: by reducing investment in productive capital the firm replenishes cash and potentially lowers other costs of risk management. The end result is higher firm value. Thus, although risk management in the long run helps mitigate underinvestment, in the short run (when w is low) it may give rise to more underinvestment. These results underscore the importance of analyzing an intertemporal model, as a dynamic analysis may reveal surprising optimal behavior that would not be plausible in a static model.

Hedging, investment, and market timing in the transitory state G . Figure 9 plots firm value, investment-capital ratio, and their sensitivities as functions of w in the transitory state G . Again, we compare the solutions with and without hedging. Note first that hedging (or speculation) significantly increases firm value (compare Panel A of Figure 8 with Panel A in Figure 9). This value gain is much larger than in the absorbing state B . Second, the marginal value of cash is lower for firms that hedge as long as the firm is not too constrained (i.e., has enough cash). For cash strapped firms the marginal value of cash is higher for firms with hedging opportunities (See Panel B of Figure 9). Third, firms issue equity later (engage in less market timing) when they hedge than when they do not (i.e., \underline{w} is lower with hedging). Similarly, firm that hedge hoard less cash and pay out to shareholders earlier.

Comparing investment policies for firms that hedge to those of firms that do not hedge we note, first, that investment is on average higher with hedging than without hedging. This follows directly from the observation that hedging increases firm value by mitigating its underinvestment problem (Froot, Scharfstein, and Stein (1993)). Second, note again that while hedging mitigates underinvestment for most values of w , it does not for sufficiently low w . The logic is the same as in the absorbing state: when w is low underinvestment is a more efficient way to manage risk.

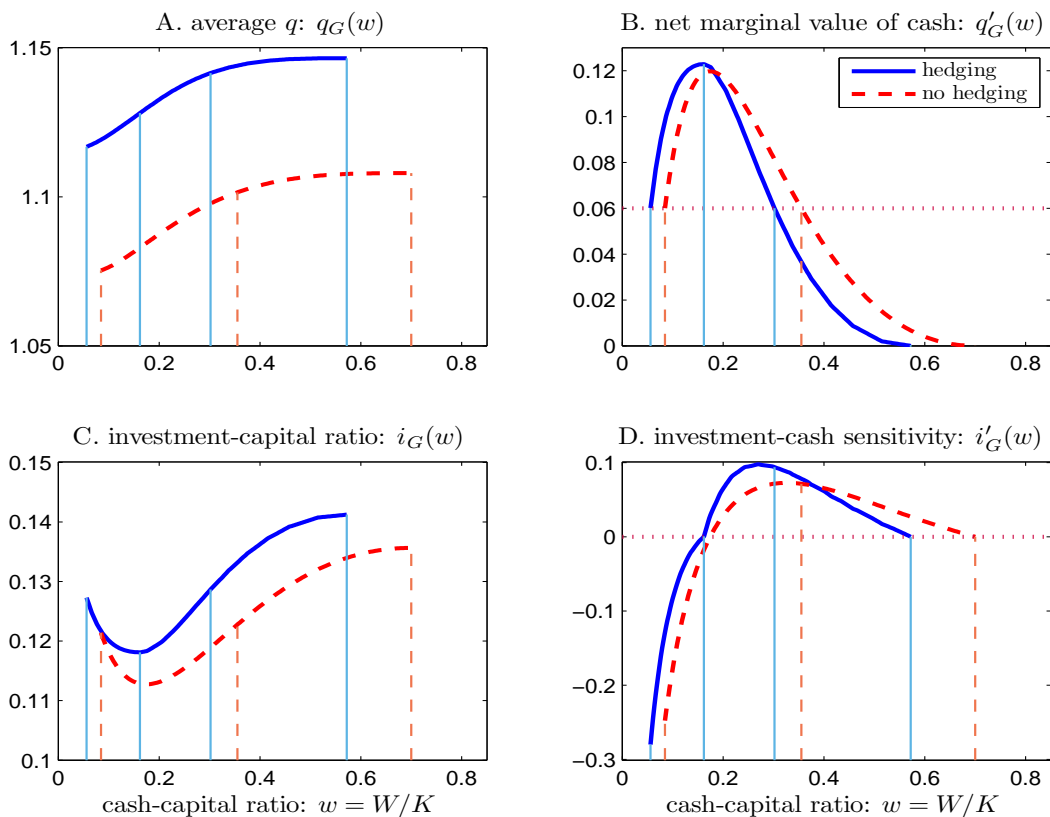


Figure 9: **Firm value and investment in the (transitory) state G .** This figure plots the average q and investment in state G with and without hedging opportunities. External financing is available in state G , but not in state B . For the hedging case, we set market volatility $\sigma_m = 20\%$, correlation coefficient $\rho = 0.6$, and margin requirements: $\pi_G = 5$ and $\pi_B = 2$. All other parameter values are given in Table 1.

8 A Recurrent Two-State Model

For expositional clarity, we have so far considered only somewhat stylized scenarios where either the “low-cost” or the “high-cost” financing state are absorbing. In reality, firms face mean-reverting financing opportunities and the reader may wonder whether our main results carry over to this more general setting. We next show that our main results on market timing, investment, and risk management continue to hold with recurrent changes in the financing conditions. Moreover, they carry over even when we allow firms to tap external equity markets in state B albeit at a high cost. We now assume that the fixed cost of financing is

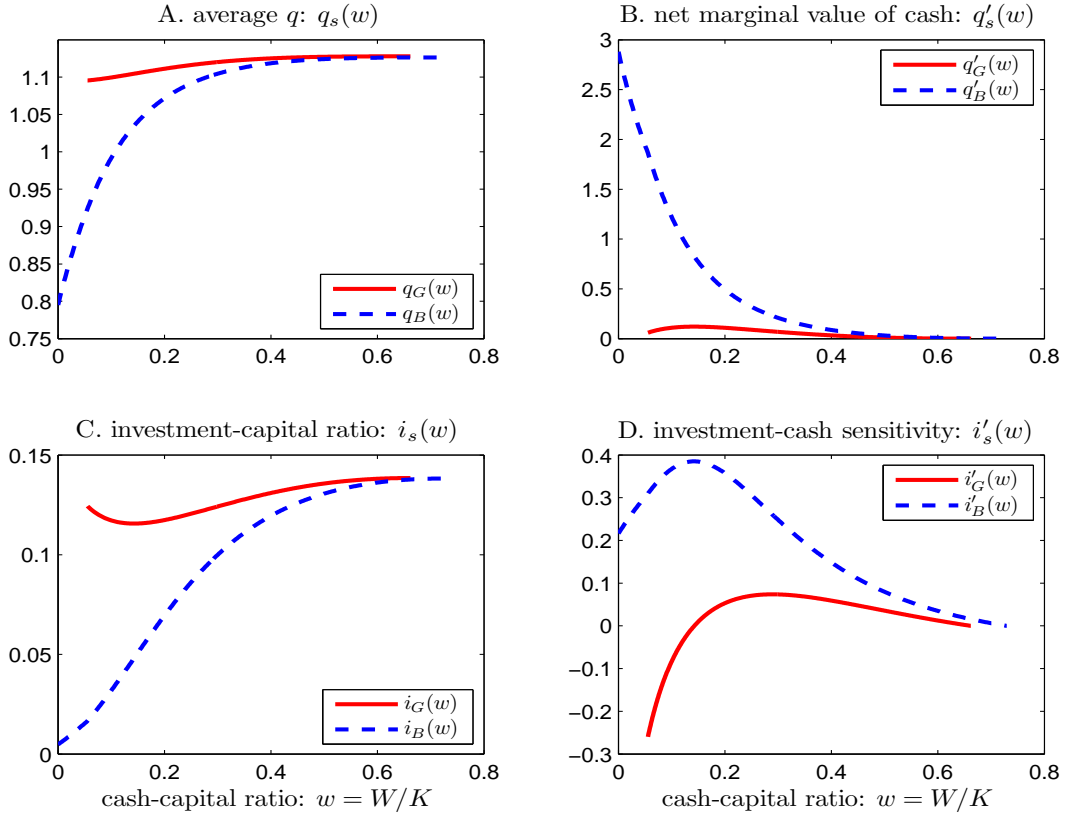


Figure 10: **Firm value and investment in the recurrent model.** This figure plots the average q and investment when the two states are recurrent. All the parameter values are given in Table 1.

$\phi_B = 30\%$, a level at which the firm still prefers financing to liquidation in state B .

Figure 10 plots firm value (average q) and investment $i_s(w)$ for both states, and their sensitivities with respect to w . The top left panel shows that average q in state G is higher than the average q in state B : the firm has higher valuation in the state with low financing cost than the state where external financing is more costly. Note that the valuation (average q) difference in this example is purely due to the difference in financing costs between the two states. Therefore, using average q to control for investment opportunities and then testing for the presence of financing constraints by using variables such as cash flows or cash (which is often done in the empirical literature) would be misleading in our setup. Second, as shown in Panel C, investment in state G is higher than in state B for a given w , but the difference

is especially large when w is low. Also, investment on average is much less variable with respect to w when external financing costs are lower.

Third, the convexity of firm value and the non-monotonicity of investment in state G for low values of w continues to hold. This is illustrated in the upper left and right panels. The intuition is essentially the same as that in the earlier sections. Financing is cheap in state G only for a finite stochastic duration, which makes the financing timing option valuable. The optimal equity issuance boundary in state G is strictly positive: $\underline{w} = 0.055$. These findings imply that our earlier results about speculation continue to hold in the recurrent setting.

Finally, as in the previous analysis, we find that in state B there is no market timing: the firm does not issue equity before it exhausts its cash holdings. Also, firm value is concave in w in state B and investment responds positively in w , as the external financing option is out of the money in state B . In sum, firm behavior in states B and G are drastically different even in the recurrent model. Financing constraints and stochastic financing opportunities significantly influence firm value (average q) and investment.

9 Conclusion

We provide a simple integrated framework of dynamic market timing, corporate investment, and risk management. Financing conditions and supply of external capital change stochastically over time. Firms anticipate the stochastic evolution of these financing opportunities and respond optimally. In particular, they optimally build war-chests by issuing equity and hoarding cash, when external financing is sufficiently cheap. For firms anticipating an equity issuance, investment may be decreasing in the firm's cash-to-asset ratio: when firms get closer to equity issuance their investment policy is less constrained by the availability of internal funds, as the firm anticipates that more cash will be raised through an equity issue in the near future. We also show that market timing is consistent with risk-seeking behavior by the firm. The key driving mechanism for these surprising dynamic implications is the finite duration of "cheap" financing conditions and the fixed cost of equity issuance.

While we provide the first dynamic framework to jointly study market timing, corporate investment, and risk management, our model is one with exogenous shifts of financing opportunities. It would clearly be desirable to consider a general equilibrium setting where the stochastic financing opportunities arise endogenously. We leave this for future research.

Table 1: SUMMARY OF KEY VARIABLES AND PARAMETERS

This table summarizes the symbols for the key variables used in the model and the parameter values in the benchmark case. For each upper-case variable in the left column (except K , A , and F), we use its lower case to denote the ratio of this variable to capital. Whenever a variable or parameter depends on the state s , we denote the dependence with a subscript s . All the boundary variables are in terms of the cash-capital ratio w_t .

Variable	Symbol	Parameters	Symbol	state G	state B
<i>A. Baseline model</i>					
Capital stock	K	Riskfree rate	r	4.34%	
Cash holding	W	Rate of depreciation	δ	15%	
Investment	I	Risk-neutral mean productivity shock	μ	21.2%	
Cumulative productivity shock	A	Volatility of productivity shock	σ	20%	
Investment adjustment cost	G	Adjustment cost parameter	θ	6.902	
Cumulative operating profit	Y	Center of adjustment cost parameter	ν	12%	
Cumulative external financing	H	Proportional cash-carrying cost	λ	1.5%	
Cumulative external financing cost	X	Proportional financing cost	γ	6%	
Cumulative payout	U	Correlation between Z_t^A and Z_t^M	ρ	0.6	
Firm value	P	Price of risk for technology shocks	η	0.4	
Average q	q				
Net marginal value of cash	q'	State transition intensity	ζ_s	0.1	0.9
Payout boundary	\bar{w}	Capital liquidation value	l_s	1.1	0.7
Financing boundary	\underline{w}	Fixed financing cost	ϕ_s	1.0%	20%
Return cash-capital ratio	m	Price of risk for financing shocks	κ_s	$\ln(3)$	$-\ln(3)$
Conditional risk premium	μ^R				
<i>B. Hedging</i>					
Hedge ratio	ψ	Market volatility	σ_m	20%	
Fraction of cash in margin account	α				
Futures price	F	Margin requirement	π_s	5	2
Maximum-hedging boundary	\hat{w}				
Speculation boundary	\tilde{w}				

Appendix

A A more general formulation of the model

Our text analysis focuses on variations of the two-state model. However, it is straightforward to generalize our model to a setting with multiple states, denoted by $s_t = 1, \dots, n$. Let the transition rate matrix for the n -state Markov chain be $\zeta = [\zeta_{ij}]$. The n -state Markov chain can capture both aggregate and firm-specific, both productivity and financing shocks (examples are business cycle or financial crises shocks). The firm's expected return on capital, volatility, and financing costs can all change when the state changes.

A.1 Risk adjustments

To properly adjust for systematic risk in the model, we assume that the economy is characterized by a stochastic discount factor (SDF) Λ_t , which evolves as

$$\frac{d\Lambda_t}{\Lambda_{t-}} = -r(s_{t-}) dt - \eta(s_{t-}) dZ_t^M + \sum_{s_t \neq s_{t-}} \left(e^{\kappa(s_{t-}, s_t)} - 1 \right) dM_t^{(s_{t-}, s_t)}, \quad (28)$$

where $r(s)$ is the risk-free rate in state s , $\eta(s)$ is the risk price for systematic Brownian shocks Z_t^M , $\kappa(i, j)$ is the relative jump size of the discount factor when the Markov chain switches from state i to state j , and $M_t^{(i,j)}$ is a compensated Poisson process with intensity ζ_{ij} ,

$$dM_t^{(i,j)} = dN_t^{(i,j)} - \zeta_{ij} dt, \quad i \neq j, \quad (29)$$

where we have utilized the result that an n -state continuous-time Markov chain with generator $[\zeta_{ij}]$ can be equivalently expressed as a sum of independent Poisson processes $N_t^{(i,j)}$ ($i \neq j$) with intensity parameters ζ_{ij} (see e.g., Chen (2010)).²² The above SDF captures two

²²More specifically, the process s solves the following stochastic differential equation, $ds_t = \sum_{k \neq s_{t-}} \delta_k(s_{t-}) dN_t^{(s_{t-}, k)}$, where $\delta_k(j) = j - i$.

different types of risk in the markets: small systematic shocks generated by the Brownian motion, and large systematic shocks from the Markov chain. We assume that dZ_t^M is partially correlated with the firm's productivity shock dZ_t^A , with instantaneous correlation ρdt . Chen (2010) shows that the SDF in (28) can be generated from a consumption-based asset pricing model.

The SDF defines a risk neutral probability measure \mathbb{Q} , under which the process for the firm's productivity shocks becomes (6). In addition, if a change of state in the Markov chain corresponds to a jump in the SDF, then the corresponding large shock also carries a risk premium, which leads to an adjustment of the transition intensity under \mathbb{Q} :

$$\hat{\zeta}_{ij} = e^{\kappa(i,j)} \zeta_{ij}, \quad i \neq j. \quad (30)$$

A.2 Solutions for the n -state model

Under the first best, the HJB equation for the n -state model can be generalized from (8) as follows,

$$r_s q_s^{FB} = \hat{\mu}_s - i_s^{FB} - \frac{1}{2} \theta_s (i_s^{FB} - \nu_s)^2 + q_s^{FB} (i_s^{FB} - \delta) + \sum_{s' \neq s} \hat{\zeta}_{ss'} (q_{s'}^{FB} - q_s^{FB}), \quad (31)$$

for each state $s = 1, \dots, n$ and the average q in state s is given by with

$$q_s^{FB} = 1 + \theta_s (i_s^{FB} - \nu_s). \quad (32)$$

While there are no closed form solutions for $n > 2$, it is straightforward to solve the system of nonlinear equations numerically.

With financial frictions, the HJB equation is generalized from (11) as follows:

$$\begin{aligned} r_s P(K, W, s) = \max_I [(r_s - \lambda_s) W + \hat{\mu}_s K - I - \Gamma(I, K, s)] P_W(K, W, s) + \frac{\sigma_s^2 K^2}{2} P_{WW}(K, W, s) \\ + (I - \delta K) P_K(K, W, s) + \sum_{s' \neq s} \hat{\zeta}_{ss'} (P(K, W, s') - P(K, W, s)), \end{aligned} \quad (33)$$

for each state $s = 1, \dots, n$, and $\underline{W}_s \leq W \leq \overline{W}_s$. We conjecture that firm value is homogeneous of degree one in W and K in each state, so that

$$P(K, W, s) = p_s(w)K, \quad (34)$$

where $p_s(w)$ solves the following system of ODE:

$$\begin{aligned} r_s p_s(w) = \max_{i_s} [(r_s - \lambda_s) w + \hat{\mu}_s - i_s - g_s(i_s)] p'_s(w) + \frac{\sigma_s^2}{2} p''_s(w) \\ + (i_s - \delta) (p_s(w) - w p'_s(w)) + \sum_{s' \neq s} \hat{\zeta}_{ss'} (p_{s'}(w) - p_s(w)). \end{aligned} \quad (35)$$

The boundary conditions in each state s are defined in similar ways as in Equation (14-21).

B Beta Representation

As indicated by the SDF Λ_t in (28) with $n = 2$, in state s , the price of risk for technology shock (risk premium for a unit exposure to the shocks) is $\lambda_s^T = \eta_s$, whereas the price of risk for financing shock is $\lambda_s^F = -(e^{\kappa_s} - 1)$. Thus, we can rewrite the risk premium using the Beta representation:

$$\mu_s^R(w) = \beta_s^T(w) \lambda_s^T + \beta_s^F(w) \lambda_s^F, \quad (36)$$

where

$$\beta_s^T(w) = \rho_s \sigma_s \frac{p'_s(w)}{p_s(w)} \quad (37)$$

$$\beta_s^F(w) = \zeta_s \frac{p_{s^-}(w) - p_s(w)}{p_s(w)} \quad (38)$$

are the technology Beta and financing Beta respectively for the firm in state s . The technology Beta will be large when the marginal value of cash relative to firm value is high; the financing Beta will be large when the probability that the financing condition will change is

high, or when the change in financing condition has large impact on the firm value.

Since there are two sources of aggregate shocks in this model, the CAPM does not hold. Instead, expected returns can be explained by a two-factor model. We assume that there are two diversified portfolios T and F , each only subject to one type of aggregate shocks, i.e. technology shocks or financing shocks, respectively. Suppose their return dynamics are as follows:

$$dR_t^T = (r_s + \mu_s^T)dt + \sigma_s^T dB_t, \quad (39)$$

$$dR_t^F = (r_s + \mu_s^F)dt + \left(e^{\kappa_s^F} - 1\right) dM_t^1 + \left(e^{\kappa_s^F} - 1\right) dM_t^2. \quad (40)$$

Then, the stochastic discount factor (28) implies that

$$\mu_s^T = \sigma_s^T \eta_s, \quad (41)$$

$$\mu_s^F = \zeta_s (e^{\kappa_s^F} - 1) (e^{\kappa_s} - 1). \quad (42)$$

We can now rewrite the risk premium in (39) and (40) using Betas as follows:

$$\mu_s^R(w) = \beta_s^T(w) \mu_s^T + \beta_s^F(w) \mu_s^F, \quad (43)$$

where

$$\beta_s^T(w) = \frac{\rho_s \sigma_s p'_s(w)}{\sigma_s^T p_s(w)} \quad (44)$$

$$\beta_s^F(w) = \frac{p_{s^-}(w) - p_s(w)}{p_s(w) (e^{\kappa_s^F} - 1)} \quad (45)$$

are the technology Beta (Beta with respect to Portfolio T) and financing Beta (Beta with respect to Portfolio F) for the firm in state s . The technology Beta will be large when the marginal value of cash relative to firm value is high; the financing Beta will be large when the probability that the financing condition will change is high, or when the change in financing condition has large impact on the firm value.

C Dynamic Hedging

We now derive the optimal hedging policy in detail for Section 7. The firm's cash holding thus evolves as follows:

$$dW_t = K_t [\mu(s_t)dA_t + \sigma(s_t)dZ_t] - (I_t + \Gamma_t) dt + dH_t - dU_t + [r(s_t) - \lambda(s_t)] W_t dt + \psi_t W_t \sigma_m dB_t, \quad (46)$$

where $|\psi_t| \leq \pi(s_t)$. To avoid unnecessary repetition, we only consider the case with positive correlation, i.e., $\rho > 0$. We consider the more interesting case where the absorbing state is the crisis state and the firm is currently in the transitory state G . We first summarize the risk management rules in the absorbing state, effectively the results from BCW. Then, we analyze the hedge ratio in the transitory state G .

In the absorbing state B . After reaching the absorbing state, the firm faces the same decision problem as the firm in BCW does. For simplicity, in the crisis state, as in the previous section, the firm has no external financing but can enter index futures contract.

BCW show that the optimal hedge ratio (with time-invariant opportunities) is given by

$$\psi_B^*(w) = \max \left\{ \frac{-\rho\sigma_B}{w\sigma_m}, -\pi_B \right\}. \quad (47)$$

Intuitively, the firm chooses the hedge ratio ψ so that the firm only faces idiosyncratic volatility after hedging. The hedge ratio that achieves this objective is $-\rho\sigma_B\sigma_m^{-1}/w$. However, this hedge ratio may not be attainable due to the margin requirement. In that case, the firm chooses the maximally admissible hedge ratio $\psi_B^*(w) = -\pi_B$. Equation (47) captures the effect of margin constraints on hedging. Because there is no hair cut (i.e., $\epsilon = 0$), the hedge ratio ψ given in (47) is independent of firm value and only depends on w . We next turn to the focus of this section: hedging in the transitory state G .

In the transitory state G . Before entering the crisis state, the firm has external financing opportunity. Moreover, the margin requirement may be different (i.e., $\pi_G > \pi_B$). In the

transitory state G , the firm chooses its investment policy I and its index futures position ψW to maximize firm value $P(K, W, G)$ by solving the following HJB equation:

$$r_G P(K, W, G) = \max_{I, \psi} [(r_G - \lambda_G) W + \mu_G K - I - \Gamma(I, K, G)] P_W + (I - \delta K) P_K \quad (48)$$

$$+ \frac{1}{2} (\sigma_G^2 K^2 + \psi^2 \sigma_m^2 W^2 + 2\rho \sigma_m \sigma_G \psi W K) P_{WW} + \zeta [P(K, W, G) - P(K, W, B)] ,$$

subject to $|\psi| \leq \pi_G$.

When firm value is concave in cash (i.e., $P_{WW}(K, W, G) < 0$), we have the same solution as in the absorbing state with margin π_G , i.e. $\psi_G^*(w) = \max \{-\rho \sigma_G \sigma_m^{-1} / w, -\pi_G\}$. However, market timing opportunities combined with fixed costs of equity issuance imply that firm value may be convex in cash, i.e., $P_{WW}(K, W, G) > 0$ for certain regions of $w = W/K$. With convexity, the firm naturally *speculates* in derivatives markets. Given the margin requirement, the firm takes the maximally allowed futures position, i.e. the corner solution $\psi_G(w) = \pi_G$. Note that the firm is long in futures despite positive correlation between its productivity shock and the index futures. Let \hat{w}_G denote the endogenously chosen point at which $P_{WW}(K, W, G) = 0$, or $p_G''(\hat{w}_G) = 0$. We now summarize the firm's futures position in the transitory state as follows:

$$\psi_G^*(w) = \begin{cases} \max \{-\rho \sigma_G \sigma_m^{-1} / w, -\pi_G\}, & \text{for } w \geq \hat{w}_G, \\ \pi_G, & \text{for } \underline{w}_G \leq w \leq \hat{w}_G. \end{cases} \quad (49)$$

Note the discontinuity of the hedge ratio $\psi_G^*(w)$ in w . The firm switches from a hedger to a speculator when its cash-capital ratio w falls below \hat{w}_G .

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