# Investment Shocks and Asset Prices: An Investment-based Approach<sup>\*</sup>

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July 2, 2019

Accepted by Journal of Financial and Quantitative Analysis

\*We thank Hendrik Bessembinder (the editor), an anonymous referee, Jack Favilukis, Vito Gala, João Gomes, Alessandro Graniero, Haibo Jiang, Jiri Knesl, Jun Li, Erik Loualiche, Georgios Skoulakis, Amir Yaron, Harold Zhang, and seminar participants at Boston University, Cheung Kong Graduate School of Business, Central University of Finance and Economics, Chinese University of Hong Kong (Shenzhen), Fudan University, Renmin University of China, University of Arizona, University of British Columbia, University of Pennsylvania (Wharton), and conference participants at 2016 China International Conference in Finance, 2016 Summer Institute of Finance, SFS Cavalcade North America 2018, and the 2019 BI-SHoF Conference for valuable comments and discussions. We are especially grateful to Ryan Israelsen for sharing with us the quality-adjusted equipment price series.

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### Abstract

We propose a new approach, based on investment data, to determine firms' return exposure to investment-specific technology (IST) shocks. When applied to U.S. data, we find that, in contrast to the pattern estimated from empirical IST proxies, value firms have *higher* exposure to IST shocks than growth firms. When applied to simulated data from existing theoretical models, our approach reveals that existing empirical findings may result from measurement errors in the IST proxies. Importantly, our simulation analysis uncovers the key role played by investment data in determining the economic mechanism through which IST shocks affect cross-sectional asset prices.

JEL classification: E22; G12; O30 Keywords: Investment shocks; Capital expenditures; Risk exposure

# 1 Introduction

Capital-embodied, investment-specific technology shocks (hereafter referred to as "investment shocks" or "IST shocks") are technological innovations that materialize through the creation of new capital stock. Since the work of Solow (1960), these investment shocks have been recognized as an important determinant of economic growth and business cycle fluctuations. More recently, financial economists have stressed their importance for explaining cross-sectional and time-series properties of returns. Unfortunately, IST shocks are not directly observable and are commonly measured through noisy proxies constructed from either macroeconomic or financial data. Existing studies find that proxies built on macro data typically exhibit correlations close to zero with proxies built on financial data, suggesting that our understanding of the effects of IST shocks on asset prices could be undermined by mis-measurement of IST shocks.<sup>1</sup>

In this paper we propose a novel approach to construct firms' return exposure to IST shocks, that, unlike existing studies, does not rely on noisy empirical proxies of IST shocks. Our approach rests on the idea that IST shocks affect firm value directly through the cost of firms' expected investment. Because the change of a firm's market value in response to an IST shock is proportional to the firm's expected investment, the *percentage* change, that is the exposure of a firm's *return* to an IST shock, can be measured by the firm's expected investment expenditures *relative* to its market value.

A simple example illustrates the key intuition behind our investment-based measure of IST risk exposure. Consider a firm with a market value of P that plans to invest one-time amount of capital I. Suppose that the occurrence of an IST shock  $\varepsilon$  decreases the unit price of capital from 1 to  $1 - \varepsilon$ . As a result, upon the occurrence of the shock, the firm value increases by the saving in investment costs, i.e.,  $\Delta P = I \times \varepsilon$ , which implies a realized return  $r = \Delta P/P = (I/P) \times \varepsilon$ . The firm's return IST-beta, by definition, is given by  $\beta^{\text{IST}} = \text{cov}(r, \varepsilon)/\text{var}(\varepsilon) = I/P$ . Therefore, we can estimate the firm's exposure to IST shocks directly from its investment-to-price ratio without relying on the empirical measure of IST shock  $\varepsilon$ . As we show in Section 2, this intuition generalizes to the case of multi-period investment in which the firm's IST-beta is determined by the ratio of present value of future investment to current equity value.

<sup>&</sup>lt;sup>1</sup>For example, Garlappi and Song (2017a) find that the correlation between two commonly used IST proxies the change in the relative price of equipment and the return spread between investment and consumption goods producers—is only 0.03 in the 1930–2012 period and 0.02 in the more recent 1964–2012 period.

We use our approach to estimate investment-based IST betas of book-to-market portfolios from a sample of U.S.-listed common stocks from 1963 to 2016. This analysis unveils a potentially different interpretation of the economic mechanism underlying the pricing effect of IST shocks from that available in the existing literature. Specifically, we find that investment-based IST betas are monotonically increasing with book-to-market ratio, with value firms exhibiting much *higher* exposure to IST shocks than growth firms. This pattern is in stark contrast with that of proxy-based IST betas. For example, using two commonly used proxies for IST shocks, one based on macro price data and one based on financial return data, we find that, proxy-based IST betas are either hump-shaped or V-shaped in the book-to-market ratio, with value firms exhibit either *similar* or *lower* exposure to IST shocks than growth firms. Our main empirical findings on investment-based IST betas of book-to-market portfolios are robust to (i) different choices of parameters in the implementation of empirical IST betas; (ii) different assumptions regarding the portion of the capital expenditures that are directly affected by IST shocks; (iii) different ways of treating firms with missing observations; and (iv) different sample of firms.

Although our investment-based approach can be used to estimate the IST-beta of any portfolio of stocks, in this paper we focus on the cross section of book-to-market sorted portfolios. This choice is motivated by the following reasons. First, the existing literature argues that, because value firms have lower investment rates, I/K, they should have lower IST-betas. In contrast, our investment-based approach implies that IST-betas are proportional to I/P ratio. Therefore, in order to test these two competing views, it is important to choose a cross section of portfolios for which these two investment ratios differ. The cross section of book-to-market portfolios is a natural choice, because sorting on B/M is equivalent on sorting on the relative value of the two ratios: B/M = K/P = (I/P)/(I/K). Second, even though IST shocks have been studied in many other cross-sections such as past investment, profitability, market beta, or idiosyncratic volatility, the economic mechanism linking IST shocks to returns is most transparent for book-to-market portfolios (see, e.g., Kogan and Papanikolaou (2013, 2014)). Finally, book-to-market portfolios are arguably one of the most extensively studied cross-sections in the asset pricing literature.

To further understand our empirical findings on the IST-betas of book-to-market portfolios and link them to the existing literature, we also apply our investment-based approach to simulated data obtained from the well-known theoretical framework of Kogan and Papanikolaou (2013, 2014). In this framework, the exposure of stock returns to IST shocks is given by the relative weight of growth opportunities in a firm's value. Because a firm's growth opportunities are not observable, empirical proxies of IST shocks are required to estimate the IST betas. We show that, within such a framework, IST betas can be computed *directly* as the ratio of a firm's expected discounted future investment expenditures to its market value. That is, our investment-based IST beta and the proxy-based beta are *equivalent* within such a model. This makes the model an ideal economic environment for understanding our empirical findings.

Using simulated data from the model, we show that, when IST shocks are measured with error, the patterns of proxy-based IST betas across book-to-market portfolio can be fragile, and this might explain the discrepancy between proxy-based and investment-based IST betas in the data. We further show that, within the structure of the simulated model, when cross-sectional variation in the book-to-market ratio is driven primarily by variation in growth opportunities, growth firms tend to have a *higher* exposure to IST shocks as it is commonly assumed in the existing literature. However, when cross-sectional variation in the book-to-market ratio is driven primarily by variation in the value of assets-in-place, growth firms have a *lower* exposure to IST shocks. We emphasize that these results hold *within* the theoretical framework of Kogan and Papanikolaou (2013, 2014). Our goal is not to assess whether this framework is the correct framework to analyze the effect of IST shocks on asset prices but to reconcile, in a controlled experiment environment, the discrepancy between investment-based and proxy-based IST beta documented in our empirical work.

Our analysis highlights the importance of investment data in understanding the impact of IST shocks on asset prices. In particular, investment data not only provide an alternative way to estimate IST beta, but can also help in discerning between competing economic interpretation of the data. For example, in the context of the aforementioned structural framework, to match the empirical fact that investment-to-price ratio increases in book to market, one would need the value of assets-in-place, instead of the growth opportunities, to be the key driver of cross-sectional variation in the book-to-market ratio. Within the model, this requirement implies that growth firms have a *lower* exposure to IST shocks. In addition, because in the data investment-based IST exposure increases in the book-to-market ratio, IST shocks can help explain the observed

value premium only if the implied IST risk premium is *positive*. Again, we emphasize that this conclusion hold *within* the simulated model, in which IST-exposures are the main driving force of cross-sectional return difference. Importantly, we do not claim that exposures to IST shocks *alone* can explain the cross-section of returns.<sup>2</sup> Instead, we provide an alternative way to measure IST-betas that can be useful in future studies featuring the link between cross-sectional returns and investment shocks.

A growing literature in macroeconomics and finance studies the effect of IST shocks on growth, business cycles, and asset prices. Greenwood, Hercowitz, and Krusell (1997, 2000) and Fisher (2006) show that IST shocks can account for a large fraction of growth and variations in output and Justiniano, Primiceri, and Tambalotti (2010) emphasize the importance of IST shocks for business cycles. Christiano and Fisher (2003) study the implications of IST shocks for aggregate asset prices. Papanikolaou (2011) introduces IST shocks in a two-sector general equilibrium model. Garlappi and Song (2017b) emphasize the importance of capital utilization flexibility and product market competition in determining the equilibrium effects of IST shocks on asset prices.

Our paper is closely related to more recent studies that investigate the pricing impact of IST shocks on cross-sectional asset returns. Kogan and Papanikolaou (2013, 2014) explore how IST shocks can explain return patterns in the cross-section that are associated with firm characteristics. Yang (2013) uses investment shocks to explain the commodity basis spread. Garlappi and Song (2017a) use proxy-based measures of firms' IST exposure to assess the ability of IST shocks to explain the magnitude of the value premium and momentum profits in the U.S. stock market. Li (2018) proposes a rational explanation of the momentum effect in the cross-section by using investment shocks. Dissanayake, Watanabe, and Watanabe (2017) provide international evidence on the effect of IST shocks on asset returns. We complement this literature by providing a new, investment-based, methodology to estimate the IST risk exposure and analyze its implications for understanding the existing economic mechanisms linking IST shocks to asset prices.

 $<sup>^{2}</sup>$ It is certainly possible that, in the real world, there are aggregate shocks other than IST, and that the exposures to these shocks are correlated with IST-beta. In this case, we cannot easily identify the risk premium related to IST shocks without explicitly accounting for these additional shocks.

Our work also relates to the large investment-based asset pricing literature that emphasizes the link between investment and stock returns. This literature explores the role of firms' optimal investment decisions in the determination of expected stock returns—see, e.g., Cochrane (1991, 1996), Liu, Whited, and Zhang (2009), and Lin and Zhang (2013). Similarly, our main idea is to exploit firms' investment data to infer their return exposure to capital-embodied technical change.

Finally, our paper is also relates to a vast literature, pioneered by Berk, Green, and Naik (1999), that uses structural models of heterogeneity in firms' investment decisions to study the cross section of returns. Significant contributions include Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), and Zhang (2005). Recent studies that introduce sources of risk in addition to neutral productivity shocks include Garleanu, Kogan, and Panageas (2012) and Garleanu, Panageas, and Yu (2012). Our paper complements this literature by studying the potential role of IST shocks in explaining the cross-section of asset returns.

Our paper makes three contributions to the asset pricing literature. First, we provide a new, theoretically motivated, methodology to study the effect of IST shocks on asset prices that does not require the use of potentially misspecified proxies of IST shocks. Second, we provide new, independent, evidence on the relative risk exposures to IST shocks for book-to-market portfolios. Finally, we uncover the key role of investment data in studying the economic mechanism through which IST shocks affect asset prices.

The rest of the paper is organized as follows. Section 2 provides the general idea and implementation of our investment-based approach to measuring IST betas. Section 3 documents empirical evidence on IST betas for book-to-market portfolios. Section 4 applies our approach to the simulated data from an existing structural model of investment. Section 5 discusses the implications of our analysis and Section 6 concludes.

# 2 The investment-based approach to estimate IST exposure

In this section we first develop the general idea underlying the use of investment expenditures to measure a firm's return exposure to IST shocks. We then show how to empirically implement this measure.

## 2.1 The general idea

Our approach to construct a firm's risk exposure to IST shocks rests on a simple, intuitive, idea: since investment shocks affect firms through the cost of investment, investment expenditures should be a key ingredient in the estimation of firms' risk exposure to investment shocks.

To formalize this intuition, let us consider an infinitely-lived firm that produces output through a declining-return-to-scale technology requiring physical capital as the only input. In each period, the firm decides whether to incur investment expenditures in order to increase capital. The firm value is equal to the present value of future net cash flows, which are determined by output net of investment expenditures. The price of new capital is subject to exogenous shocks, which we refer to as IST shocks.



Figure 1: IST shocks and investment expenditures: a graphical illustration.

Figure 1 provides a graphical illustration of the proposed measure of a firm's exposure to an IST shock. In the figure we consider a firm's optimal choice of physical capital. The declining curve Q represents the marginal value of capital. The horizontal line  $p^{I}$  represents the marginal cost of investment, i.e., the price of new capital, which, for simplicity, we take as constant.<sup>3</sup> Let us consider an IST shock  $\varepsilon$  to the price of capital. We assume that a positive IST shock  $\varepsilon$  causes a drop in the marginal cost of capital from  $p^{I}$  to  $p^{I^*} = (1 - \varepsilon) \times p^{I}$ , but does not affect the marginal value of capital Q.

<sup>&</sup>lt;sup>3</sup>The main intuition is unaffected by considering an increasing marginal cost function as in the case of convex capital adjustment costs.

As a consequence of the shock  $\varepsilon$ , the firm will "save" on investment costs, and hence increase its NPV by an amount represented by the shaded area in Figure 1, which is approximately equal to

$$\Delta NPV \approx \varepsilon \times p^{I} \times K^{*} = \varepsilon \times \frac{p^{I^{*}}}{1 - \varepsilon} \times K^{*} \approx \varepsilon \times p^{I^{*}} \times K^{*} = \varepsilon \times I^{*}, \tag{1}$$

where we ignore terms of order  $o(\varepsilon^2)$  and use the fact that investment expenditure  $I^* = p^{I^*} \times K^{*,4}$ Hence, *per unit of shock*  $\varepsilon$ , the NPV increases "on impact" by an amount  $I^*$  and this positively affects the firm's value. If the IST shock is persistent, it impacts not only the current period but also all future investment costs. Therefore, the effect of an IST shock at time t on firm value can be written approximately as follows

$$\Delta P_t \equiv P_t - P_{t-1} \approx \varepsilon \times PV_t \left( \sum_{s=0}^{\infty} I_{t+s}^* \right), \tag{2}$$

where  $P_t$  is the firm's market value at time t,  $I_{t+s}^*$  is the investment expenditure at time t + s, and  $PV_t(\cdot)$  denotes present value at time t. The firm's return beta on the IST shock is then

$$\beta_t^{\text{IST}} = \frac{cov(\Delta P_t/P_{t-1}, \epsilon)}{var(\epsilon)} \approx \frac{PV_t\left(\sum_{s=0}^{\infty} I_{t+s}^*\right)}{P_{t-1}}.$$
(3)

Equation (3) illustrates that, in this simple framework, investment expenditures are directly related to a firm's return sensitivity to IST shock. The expression for  $\beta_t^{\text{IST}}$  is intuitive: a persistent per-unit positive IST shock decreases all future investment cost, and therefore increases firm value by the discounted future investment expenditures, that is,  $PV_t\left(\sum_{s=0}^{\infty} I_{t+s}^*\right)$ . The increase in firm value scaled by lagged firm value,  $P_{t-1}$ , represents the response of the firm's *return* to the IST shock, that is, its IST beta.

It is important to emphasize that the above argument does not require investment expenditures to be driven *only* by IST shocks. By definition, an IST shock is represented by a change in the marginal cost of capital. Any other shock that is orthogonal to the IST shock will affect the marginal value of capital. In the context of the model illustration in Figure 1, an IST shock *is* a change in the capital good price  $p^{I}$  while any other shock impacts the marginal value of

<sup>&</sup>lt;sup>4</sup>Alternatively, we could have used the approximation  $\widehat{\Delta NPV} \approx \varepsilon \times p^I \times K = \varepsilon \times I$ , with  $I = p^I \times K$ . Because  $K^* - K$  is of order  $\varepsilon$ , the difference between  $\widehat{\Delta NPV}$  and  $\Delta NPV$  in equation (1) is of order  $o(\varepsilon^2)$ . Note, however, that empirically we observe only the investment response to the IST shock,  $I^*$ , but not I.

capital Q. Investment expenditures are therefore determined jointly by all shocks affecting both the price and the marginal value of capital. The argument leading to equation (3) holds for *any* given marginal value of capital Q. Hence, the investment-based beta in equation (3) captures firms' return sensitivity to IST shocks, even if investment expenditures are driven also by other shocks orthogonal to IST shocks. In Section 4.1 we will confirm this argument within the context of a structural model of investment with two shocks.

Note finally that the investment-based IST beta in equation (3) only captures the partial equilibrium effect of IST shocks on the cost of investment but not the general equilibrium effects on the value of existing assets. In order to assess such effects, one has to commit to a specific structure of preferences and technology, implying that any inference will be sensitive to these specific choices (see, e.g., Papanikolaou (2011) and Garlappi and Song (2017b)). The partial equilibrium approach that we take in this paper is however useful for studying the effect of IST shocks on asset prices, as shown in prior work (see, e.g., Kogan and Papanikolaou (2013, 2014), and Li (2018)).

### 2.2 Implementation

To implement the measure of IST beta derived in equation (3), we need to construct the present value of future investment expenditures. This task is equivalent to a standard valuation problem via discounted cash flows. As such, we make the same simplifying assumption typically used in a discounted cash flow implementation. First, we assume a constant positive risk premium and discount the future investment expenditures at a constant rate  $\eta$ . Second, we split the stream of future investment expenditures into two periods at the horizon T, after which we assume that investment expenditures grow at a constant rate  $g < \eta$ . Under these two assumptions, we obtain that the present value of investment expenditures is

$$PV_t\left(\sum_{s=0}^{\infty} I_{t+s}\right) = \mathbb{E}_t\left[\sum_{s=1}^{\infty} \frac{I_{t-1+s}}{(1+\eta)^{s-1}}\right]$$

$$= \mathbb{E}_t\left[\sum_{s=1}^{T-1} \frac{I_{t-1+s}}{(1+\eta)^{s-1}} + \frac{I_{t-1+T}}{\rho(1+\eta)^{T-1}}\right]$$
(4)

where  $\rho \equiv \left(\frac{\eta - g}{1 + \eta}\right)$ .

The investment-based IST-beta in equation (3) becomes

$$\beta_{T,t}^{\text{IST}} = \frac{\mathbb{E}_t \left[ \sum_{s=1}^{T-1} \frac{I_{t-1+s}}{(1+\eta)^{s-1}} + \frac{I_{t-1+T}}{\rho(1+\eta)^{T-1}} \right]}{P_{t-1}},\tag{5}$$

where T is the number of periods of investment expenditures we use in our implementation. If we further assume that the ex-post realization of investment expenditures represents a good approximation for their expected value, then we can simplify equation (5) as

$$\beta_{T,t}^{\text{IST}} = \sum_{s=1}^{T-1} \frac{1}{(1+\eta)^{s-1}} \frac{I_{t-1+s}}{P_{t-1}} + \frac{1}{\rho(1+\eta)^{T-1}} \frac{I_{t-1+T}}{P_{t-1}}.$$
(6)

Note that when T = 1, both equations (5) and (6) collapse to

$$\beta_{1,t}^{\rm IST} = \frac{1}{\rho} \frac{I_t}{P_{t-1}},\tag{7}$$

which is simply a scaled version of the investment-to-lagged-price ratio (I/P).

In our empirical implementation we only focus on IST beta at the *portfolio* level and not at the firm level. Therefore, the quantities  $I_{t+s}$  and  $P_{t-1}$  in expressions (6) and (7) refer to investment expenditures and market valuation of a portfolio of firms. Because we only consider betas at the portfolio level, ex-post realization of investment expenditures does provide a good approximation for their expected value in that idiosyncratic components of investment expenditures are diversified away. In our main analysis, we will rely on the realized investment expenditures and use (6) and (7) to estimate portfolio IST betas.

It is worth pointing out that the practical implementation of equation (6) requires knowing future investment expenditures over the next T periods. One may argue that, because investors cannot observe future investment expenditures at the current time, our approach cannot be implemented. This argument is misplaced. In fact, note that our formulation of IST-beta involving future investment makes *the same* informational assumption as any standard valuation model linking asset prices to future net cash flows, that is, the difference between output and investment. In other words, as investors have to account for future cash flow expectations in valuing stocks, so they should also account for future investment (output minus cash flow) when valuing exposure to IST shocks. Obviously, as econometricians, we do not observe investors' expectations directly but only realized investment which we use to infer expectation about future investment at the portfolio level. Alternatively, one can rely on historical data to predict future investment and use the predicted future investment in the construction of investment-based IST betas. For robustness, we explore this alternative approach in Section 3.3.3.

# 3 Analysis on empirical data

The theoretical analysis of the previous section shows that a firm's IST beta is directly related to its investment-to-price (I/P) ratio. Although, at first, it might seem intuitive to think that returns of firms with high investment rates (I/K) are more exposed to investment shocks, the above analysis points to the investment-to-price ratio I/P as the main ingredient for a conceptually correct measure of return exposure to investment shocks.

To understand the difference in the economic content of these two ratios, in our empirical analysis it is important to select a cross section of assets for which I/P and I/K ratios differ. These two ratios are linked to each other through the book-to-market ratio, that is,

$$\frac{B}{M} = \frac{K}{P} = \frac{I/P}{I/K},\tag{8}$$

Therefore, the cross section of book-to-market sorted portfolios is a natural choice for our empirical analysis, because sorting on the B/M is equivalent to sorting on the relative value of the two investment ratios.

We emphasize that, our empirical analysis is grounded in the theory developed in Section 2.1, showing that IST-beta are *directly* related to the I/P ratio. This investment-based approach to IST-beta is independent of our choice of the specific cross-section. In other words, one can apply the measures constructed in Section 2.2 to *any* cross-section of portfolios, including B/M-sorted portfolios. The focus of our empirical analysis is to estimate the IST-betas of B/M portfolios based on investment data and compare them with those based on IST proxies.

We consider all U.S. common stocks (with share code of 10 or 11) from 1963 to 2016. Price and return data are from CRSP, and accounting data are from Compustat. Because our focus is on firms' investment in capital goods, we exclude financial stocks, that is, firms with Standard Industry Classification (SIC) codes between 6000 and 6999. At the end of each year, we first sort firms into ten portfolios, according to their book-to-market ratio and then keep track of firms in each portfolio over subsequent years, as described in more details in Appendix  $B^{5}$ .

As discussed in Section 2.2 (see equations (6) and (7)), the implementation of the investmentbased IST betas depends on the two free parameters:  $\rho$  and  $\eta$ , representing the discount rates for future investment expenditures. In our benchmark analysis, we choose  $\eta = 12\%$ , which is roughly the same as the average market return from 1963 to 2016. To guarantee that  $\beta_{1,t}^{\text{IST}}$  and  $\beta_{T,t}^{\text{IST}}$  have comparable magnitudes, we chose a value of  $\rho$  equal to 4%. Due to data limitations, when implementing  $\beta_{T,t}^{\text{IST}}$  from equation (6) we take T = 15 as our benchmark measure. In Section 3.3, we assess the robustness of our parameter choice to different values of the discount rates and investment horizon T.

#### 3.1 Investment-based betas of book-to-market portfolios

Because our measures of IST-beta depend on investment data, we first analyze the investment patterns of B/M portfolios after portfolio formation. As measures of investment (I) and capital (K), we use, respectively, firm-level capital expenditures (CAPX) and property, plant, and equipment-total (PPENT) from COMPUSTAT. As a measure of market value (P), we use market capitalization from CRSP. Following the definition of  $\beta_{T,t}^{\text{IST}}$  in equation (6), we keep track of firms in each portfolio for T = 15 years and compute two investment-related ratios. The first ratio is the investment rate, defined as  $I/K \equiv I_{t+s-1}/K_{t+s-2}$ , with  $s = 1, \ldots, 15$ . The second ratio is the investment-to-price ratio, which, following equation (6) we define as  $I/P \equiv \frac{1}{(1+\eta)^{s-1}}I_{t+s-1}/P_{t-1}$ , for  $s = 1, \ldots, 15$ .<sup>6</sup> We provide more details on the construction of portfolio-level ratios in Appendix B.

Table 1 reports the I/K ratios (Panel A) and I/P ratios (Panel B) of B/M portfolios over a 15-year window after portfolio formation. In the first year after formation, s = 1, the I/Kratio is monotonically decreasing in the B/M ratio. The spread in I/K ratios between the value and growth portfolios (HML) is -18% in the first year after portfolio formation. This spread in

 $<sup>^{5}</sup>$  To control for industry effects, in an unreported robustness analysis, we form book-to-market portfolios by using within-industry sorts. We find that our main results on the investment-based IST betas still hold after removing the industry effect on book-to-market portfolios.

<sup>&</sup>lt;sup>6</sup>Specifically, we form the portfolio in year t-1, and then compute the investment to lagged capital ratio and the discounted investment-to-price ratio for the next 15 years (from year t to year t + 14).

I/K ratio shrinks to -7% after 15 years. The large spread in investment rates between value and growth portfolio after portfolio formation motivates the conjecture that growth firms should have higher exposure to investment shocks.

Panel B reports the discounted I/P ratio of B/M portfolios over a 15-years window. In stark contrast with the I/K ratio in Panel A, in the first year after portfolio formation, the I/Pratio is monotonically increasing in the B/M ratio. The spread in I/P ratios between value and growth portfolio is 25% in the first year after portfolio formation. This spread decreases gradually over time but remains at a level of 15% after 15 years. According to the theoretical framework developed in Section 2, a firm's IST beta is directly linked to its discounted I/Pratio. Therefore, from the I/P ratio reported in Panel B, we infer that, contrary to common intuition, value firms have much *higher* exposure to the investment shocks than growth firms. That is, relative to their market value, value firms spend more in investment than growth firms, and their returns are therefore more exposed to IST shocks.

Table 2 displays returns and IST betas for book-to-market portfolios. Panel A reports the book-to-market portfolios' returns in excess of risk-free rate. The excess return pattern across B/M portfolios confirms the existence of a positive value premium. The return difference between the high- and low-B/M portfolios is 8.26% per year (with a *t*-value of 2.53). Panel B reports two versions of the investment-based IST betas,  $\beta_{1,t}^{\text{IST}}$  and  $\beta_{15,t}^{\text{IST}}$ , constructed according to equations (7) and (6), respectively. When we use only one year of investment data in computing IST betas,  $\beta_{1,t}^{\text{IST}}$  increases monotonically from 1.09 for the growth portfolio to 7.23 for the value portfolio, implying a spread in IST betas of 6.14 (with a *t*-value of 9.61). When we use 15 years of investment data in computing IST betas,  $\beta_{15,t}^{\text{IST}}$  also increases monotonically from 1.52 for the growth portfolio to 8.12 for the value portfolio, implying a spread in IST betas of 6.60 (with a *t*-value of 5.87). These values of IST exposure of B/M portfolios confirm the intuition from the investment ratios reported in Table 1: value firms invest more than growth firms *relative to their market capitalization*, and therefore they exhibit higher exposure to investment shocks than growth firms.

## 3.2 Comparison with proxy-based IST betas

We now compare investment-based IST betas of book-to-market portfolios to the corresponding proxy-based quantities. To construct proxy-based IST betas, we focus on two IST proxies commonly used by existing studies: one based on macroeconomic data and one based on financial market data.<sup>7</sup> The first IST proxy (*Ishock*), originally proposed by Greenwood, Hercowitz, and Krusell (1997), is defined as:

$$Ishock_{t} = -\left[\ln\left(p^{I}/p^{C}\right)_{t} - \ln\left(p^{I}/p^{C}\right)_{t-1}\right],\tag{9}$$

where  $p^{I}$  is the price deflator for equipment and software in gross private domestic investment, and  $p^{C}$  is the price deflator for nondurable consumption goods. The price deflator for nondurable consumption goods,  $p^{C}$ , is from the National Income and Product Accounts (NIPA) tables. The price deflator of investment goods,  $p^{I}$ , is obtained from the quality-adjusted series of Israelsen (2010).<sup>8</sup> A positive technological innovation reduces the relative price of new capital goods and corresponds to an increase in *Ishock*.

The second IST proxy (IMC), originally proposed by Papanikolaou (2011), is the stock return spread between investment and consumption producers, i.e.,

$$IMC_t = r_t^I - r_t^C, (10)$$

where  $r_t^I$  and  $r_t^C$  are the returns on a portfolio of firms producing, respectively, the investment and consumption goods. To determine whether a firm belongs to the investment or consumption sectors, we follow the procedure of Gomes, Kogan, and Yogo (2009) who assign each Standard Industry Classification (SIC) code to either the investment or consumption sectors on the basis of the 1987 benchmark Input-Output tables. A positive investment shock benefits investment firms relatively more and therefore results in a positive measure of IMC.

We estimate the proxy-based IST beta of a given portfolio by regressing the time series of portfolio excess returns on either: (i) an IST proxy, in univariate regressions, or (ii) an IST proxy together with the return on the market portfolio (MKT) or the growth rate in total-factor-

<sup>&</sup>lt;sup>7</sup>Other IST proxies proposed in the literature include the change in the aggregate investment to consumption ratio, and Fama and French's (1993) HML portfolio excluding investment sector firms. See, e.g., Kogan and Papanikolaou (2014).

<sup>&</sup>lt;sup>8</sup>We are grateful to Ryan Israelsen for sharing with us the extended quality-adjusted equipment price series.

productivity (TFP), in bi-variate regressions.<sup>9</sup> We repeat the estimation for all book-to-market portfolios and compute the beta spreads for value-minus-growth.

Panel C of Table 2 reports our estimates of proxy-based IST betas for book-to-market portfolios. In univariate regression, *Ishock* betas are negative for all portfolios, with most estimates statistically significant. The beta difference between value and growth portfolios is -1.41 (with a *t*-value of -1.24). The univariate *IMC* betas are all positive with most estimates statistically insignificant. The beta difference between value and growth portfolios is 0.14 (with a *t*-value of 0.44). Bivariate regressions give similar patterns of IST betas. For example, when controlling for the market factor (MKT), although *Ishock* and *IMC* betas are different from their univariate counterparts, their HML spread is similar to that observed in the univariate case. In bivariate regressions that control for TFP growth, the values of univariate and bivariate betas are very close.

The results in Panel C indicate that proxy-based IST betas are either hump-shaped or Vshaped in the book-to-market ratio with the IST betas of value firms either *similar to* or *lower* than those of growth firms. This contrasts with the patterns in Panel B, showing that investmentbased betas are monotonically increasing in the book-to-market ratio, with value firms exhibiting *higher* IST exposure than growth firms. In Section 4 we investigate the potential reasons behind the discrepancy between the two approaches by relying on simulated data from a structural model of investment.

#### 3.3 Robustness of investment-based IST betas

We explore the robustness of the patterns in the investment-based IST beta documented in Panel B of Table 2 along four dimensions: (i) the choice of time-window and discount rates used in the construction of IST betas (Section 3.3.1); (ii) the proportion of investment expenditures that are affected by IST shocks (Section 3.3.2); (iii) the treatment of missing observations (Section 3.3.3); (iv) the choice of sample (Section 3.3.4).

 $<sup>^{9}\</sup>mathrm{The}$  annual TFP is the multi-factor productivity measure for the private business sector from the Bureau of Labor Statistics.

#### **3.3.1** Alternative choice of parameters

In the benchmark analysis of Section 3.1, we choose a window of T = 15 years in the implementation of equation (6) for the construction of  $\beta_{T,t}^{\text{IST}}$ . As a robustness check, we repeat the analysis using a window of T = 20 years. Panel A of Figure 2 reports  $\beta_{1,t}^{\text{IST}}$ ,  $\beta_{15,t}^{\text{IST}}$ , and  $\beta_{20,t}^{\text{IST}}$  for the ten B/M portfolios. As the figure illustrates, all three versions of the IST-beta are monotonically increasing in B/M, and unaffected by the choice of the window T. In particular, the similarity between  $\beta_{15,t}^{\text{IST}}$  and  $\beta_{20,t}^{\text{IST}}$  indicates that our findings are robust to the choice of the cutoff year T.

In our benchmark analysis, we construct the quantities  $\beta_{1,t}^{\text{IST}}$  and  $\beta_{T,t}^{\text{IST}}$  in equations (7) and (6), respectively, by using the discount rates  $\rho = 4\%$  and  $\eta = 12\%$ . Because these discount rates apply to all B/M portfolios, different values will simply affect the magnitude of the IST-betas, but not the patterns across B/M portfolios. One might argue that different portfolios have different dynamics of investment, and therefore require different discount rates. We allow for this possibility by using the realized returns of each portfolio as a reference quantity for discount rates. Specifically, we implement equation (7) as

$$\tilde{\beta}_{i,1,t}^{\text{IST}} = \frac{I_t / P_{t-1}}{\frac{1}{3}\bar{r}_{i,t,t+14}}, \quad i = 1, \dots, 10,$$
(11)

where we replace the constant  $\rho$  with the portfolio average return  $\bar{r}_{i,t,t+14}$  in the 15-year window following portfolio formation and rescale the return by a factor of 1/3 in order to have comparable magnitude of IST-beta as in the benchmark analysis.<sup>10</sup> Note that, according to equation (11), each portfolio has a different discount rate, with the investment expenditures of the value portfolio discounted more heavily than the growth portfolio. This is a conservative choice, because, with these discount rates the IST beta spread between value and growth portfolios is smaller than with the benchmark constant discount rate.

Similarly, we implement equation (6) by discounting the stream of investment expenditures over the next T-1 years at the realized portfolio returns, and the investment in year t+T-1

<sup>&</sup>lt;sup>10</sup>In the benchmark analysis of Section 3.1, we use a constant value of  $\rho = 4\%$ . In equation (11), we replace the constant  $\rho$  by a portfolio-specific discount factor  $\rho_i = \bar{r}_{i,t,t+14}/3$ , chosen to insure that the average  $\bar{\rho}_i$  is roughly equal to the constant  $\rho$ . This rescaling of the returns guarantees that the average magnitudes of IST betas across B/M portfolios are comparable between the two alternative approaches.

at the average (rescaled) return used in equation (11). Specifically,

$$\tilde{\beta}_{i,T,t}^{\text{IST}} = \sum_{s=1}^{T-1} \frac{1}{R_{i,t,t+s-1}} \frac{I_{t+s-1}}{P_{t-1}} + \frac{1}{\frac{1}{3}\bar{r}_{i,t,t+T-1}R_{t,t+T-1}} \frac{I_{t+T-1}}{P_{t-1}}, \quad i = 1, \dots, 10,$$
(12)

where  $R_{t,t+s-1} = \prod_{j=0}^{s-1} (1 + r_{i,t+j})$ , with  $r_{i,t} \equiv 0$ , for all portfolios i = 1, ..., 10.

Panel B of Figure 2 reports  $\tilde{\beta}_{1,t}^{\text{IST}}$ ,  $\tilde{\beta}_{15,t}^{\text{IST}}$ , and  $\tilde{\beta}_{20,t}^{\text{IST}}$  for the ten B/M portfolios. The figure shows that all three versions of the IST-beta are increasing in B/M, indicating that our main analysis is robust to a different choice of implementation parameters.

## 3.3.2 Alternative assumptions on the impact of IST shocks on investment

In our baseline analysis we assume that all capital expenditures are affected by IST shocks. In reality, this might not always be the case, when, for example, firms need to incur expenses to replace parts of an aging equipment. In this case the expenditure incurred for capital replacement is likely not directly affected by the price of the state-of-the-art technology. To address this possibility, we assume that a fixed fraction  $\theta$  of a firm's installed capital  $K_{t-1}$  needs to be replaced, and that this replacement cost is not directly affected by IST shocks. Therefore, at each time t, the capital expenditures,  $\hat{I}_t$ , that are *directly* affected by IST shocks, are given by

$$\widehat{I}_t = \max\{0, I_t - \theta K_{t-1}\}.$$
(13)

Because capital expenditures are non-negative  $(I_t \ge 0)$ , our benchmark analysis corresponds to the case of  $\theta = 0$ . For robustness, we consider an alternative value for the annual depreciation parameter  $\theta = 10\%$ , which is is conservatively chosen among the higher end of the commonly used values of a firm's capital depreciation rate.<sup>11</sup>

Panel C of Figure 2 reports the investment-based IST betas constructed using the adjusted capital expenditure  $\hat{I}_t$  in equation (13). As in the benchmark case, IST betas increase monotonically with book-to-market. For example, the high-minus-low difference in  $\beta_{1,t}^{\text{IST}}$  is 1.92 with a *t*-value of 7.13. Note that the magnitudes of the betas for all three measures are lower than

<sup>&</sup>lt;sup>11</sup>If, alternatively, we assume that a fixed fraction  $\theta$  of the investment expenditures  $I_t$ , (instead of capital  $K_{t-1}$ ) is not affected by IST shocks, then  $\hat{I}_t = (1 - \theta)I_t$  and all the results in the benchmark analysis ( $\theta = 0$ ) will be essentially unaffected except for a constant rescaling factor.

the benchmark case in Panel A, since only a part of the firms' capital expenditure is subject to the investment shocks in Panel C.

Panel D of Figure 2 reports the IST betas  $\tilde{\beta}_{1,t}^{\text{IST}}$ ,  $\tilde{\beta}_{15,t}^{\text{IST}}$ , and  $\tilde{\beta}_{20,t}^{\text{IST}}$  from equations (11) and (12) constructed using the adjusted investment expenditures  $\hat{I}_t$  in (13). The overall pattern of the IST betas is unaffected: value firms have higher IST betas than growth firms.<sup>12</sup>

### 3.3.3 Alternative treatment of missing observations

As noted earlier, the construction of IST beta from equation (6) requires tracking the investment expenditures of firms in each portfolio for T years. Obviously, not all firms in a portfolio at time t will remain in our sample until time t + T. Delisting may happen for a variety of reasons, such as bankruptcy, mergers, or going private transactions. To deal with delisting, in our benchmark analysis of Section 3.1, we assume that firms with missing observations have the same investment-to-current-price ratio as those with valid investment expenditure data. In this section, we explore two alternative ways to treat missing observations.

The first alternative approach assumes that firms with missing observations make zero investment. This puts a lower bound on a portfolio's capital expenditures. Panel B of Figure 3 reports the IST betas  $\beta_{1,t}^{\text{IST}}$ ,  $\beta_{15,t}^{\text{IST}}$ , and  $\beta_{20,t}^{\text{IST}}$ , for this case. Note that all three version of the IST betas are monotonically increasing with B/M. Compared to the benchmark results reproduced in Panel A, the level of the IST betas are, not surprisingly, lower.

The second alternative approach to deal with missing observations uses *expected*, rather than realized, investment. The main idea is to use current available information to estimate a firm's expected investment. We follow Li and Wang (2017) and estimate firm i's growth rate of future investment via the following cross sectional regression:

$$\frac{I_{it}}{I_{it-1}} = b_{0,t} + b_{lag,t} \times \frac{I_{it-1}}{I_{it-2}} + b_{mom,t} \times MOM_{it-1} + b_{q,t} \times Q_{it-1} + b_{cf,t} \times CF_{it-1} + \epsilon_{it}, \quad (14)$$

where  $I_{it-1}/I_{it-2}$  denotes the one-year lagged investment growth;  $MOM_{it-1}$  is firm *i*'s cumulative stock return from January to November in the year t-1;  $Q_{it-1}$  is the log of the market value divided by net capital in year t-1; and  $CF_{it-1}$  is the sum of depreciation and income

 $<sup>^{12}</sup>$ In unreported analysis, we confirm that the investment-based IST betas have the same pattern across book-to-market portfolios if we also include R&D expenditures in the investment measure.

before extraordinary items divided by capital in year t - 1. The independent variable  $I_{it}/I_{it-1}$  is the growth rate of future investment relative to year t - 1.

To construct our measures of investment-based IST betas,  $\beta_{1,t}^{\text{IST}}$  and  $\beta_{T,t}^{\text{IST}}$ , we need the first year's investment in year t (equation (7)) and the sum of discounted investment from t to t+T-1 (equation (5)). We use the results from the regressions (14) to construct both measures of expected investment. Specifically, we first compute the time series average of the regression coefficients. We then use these averages, together with time t-1 information to predict future investment either at time t, or over the window t to t + T - 1. We finally use these expected investment estimate to construct the IST betas for  $\beta_{1,t}^{\text{IST}}$  and  $\beta_{T,t}^{\text{IST}}$  according to equation (5).

Panel C of Figure 3 reports the IST betas constructed with expected investments. Note that all three versions of the IST beta have the same pattern as in the benchmark case (Panel A). Moreover, the values of  $\beta_{1,t}^{\text{IST}}$  constructed from the expected investment closely match those constructed from realized investment (see Panels A and C). Finally, both  $\beta_{15,t}^{\text{IST}}$  and  $\beta_{20,t}^{\text{IST}}$  are higher when constructed from expected investment than when constructed from realized investment. This happens because when estimating regression (14) to estimate future *T*-year investment, we use only firms that survived over the entire *T*-year window, and therefore expected investment are inflated due to a survivorship bias.

To assess the importance of this survivorship bias, we re-estimate regression (14) by assuming that firms with missing observations make zero investment. In this case, we retain all firms in the cross section, even those that have less than T years of investment data. This approach mimics the way we deal with missing observations in Panel B. Panel D shows that the betas obtained using this procedure are qualitatively similar to those in the benchmark case reported in Panel A. Note finally that the magnitude of betas in Panels B and D are comparable, indicating that the expected investment model (14) provides a fairly accurate estimate of actual portfolio investment.

#### 3.3.4 Alternative samples

The analysis so far has considered a sample of non-financial firms. In this section, we assess the robustness of our main findings to two alternative samples of firms. First, we consider a sample of non-financial and consumption firms, excluding investment firms. IST shocks are likely to affect investment firms through both their cost of investment and their value of output. Therefore, by excluding investment firms, we mitigate the concern that investment expenditures may only capture a partial effect of IST shocks on those firms. Second, we consider a sample of non-financial firms that have fiscal year end in December. The December fiscal year end helps to align the accounting and calendar time, eliminating potential mismatch in the timing of information.

Panel A of Figure 4 reproduces the benchmark results for non-financial firms. Panel B reports IST betas for non-financial and consumption firms. Panel C reports IST betas for non-financial firms with a December fiscal year end. Finally, Panel D reports IST betas of non-financial and consumption firms with a December fiscal year end. All these different samples give both qualitatively and quantitatively similar patterns of IST betas across B/M portfolios.

To summarize, the robustness analysis performed in this section and the evidence reported above indicate that the IST beta spread between value and growth is *positive*. That is, our investment-based approach suggests that value firms have *higher* exposure to investment shocks than growth firms. This finding is robust to a wide range of empirical specifications.

# 4 Analysis on simulated data

To understand the discrepancy between investment-based and proxy-based approach to construct IST betas in the data, we apply both approaches to simulated data from a structural model of investment. In particular, we specialize the general framework introduced in Section 2 to the partial equilibrium model of firm investment with vintage capital of Kogan and Papanikolaou (2014, KP hereafter).<sup>13</sup> This allows us to (i) validate the empirical implementation of the investment-based measure of IST exposure in equations (6) and (7), and (ii) compare *analytically* investment-based and proxy-based IST betas within the framework of the selected structural model. More important, this also allows us to carry out economic experiments within the model

<sup>&</sup>lt;sup>13</sup>The modelling framework in Kogan and Papanikolaou (2014) is similar to that in Kogan and Papanikolaou (2013) with two main differences. First, Kogan and Papanikolaou (2014) consider firm-specific idiosyncratic productivity shocks, while Kogan and Papanikolaou (2013) do not. The absence of firm-specific shocks weakens the relationship between profitability of a firm's existing assets and its growth opportunities. Second, Kogan and Papanikolaou (2013) introduce uncertainty about the firm's growth opportunities, which can be learned from a public signal. This learning feature helps to link growth opportunities to idiosyncratic volatility. For simplicity, we follow closely the setup in Kogan and Papanikolaou (2014).

which can help us to better understand not only our earlier empirical findings in Section 3, but also the economic mechanisms through which IST shocks affect firms' values and returns.

## 4.1 A structural model of investment

KP consider a continuum  $\mathcal{F}$  of measure one of infinitely lived firms who behave competitively in the product market but have monopoly access to their growth opportunities. At time t, each firm  $f \in \mathcal{F}$  owns a finite number  $\mathcal{J}_t^f$  of existing projects. Project j, owned by firm f, produces a flow of output equal to

$$y_{fjt} = \varepsilon_{ft} u_{jt} x_t K_j^{\alpha}, \tag{15}$$

where  $\varepsilon_{ft}$  a firm-specific productivity shock;  $u_{jt}$  is a project-specific productivity shock;  $x_t$  is the common productivity shock for all existing projects;  $K_j$  is the project's physical capital which is determined at the time of the project's initial investment; and,  $\alpha \in (0, 1)$  captures decreasing returns to scale at the project level. Each project expires independently at a Poisson death rate  $\delta$ . Given these assumptions, capital in the model is not homogenous but stratified across different "vintages", depending on the active projects within the firm.

The evolution of the three shocks is governed by the following processes:

$$d\varepsilon_{ft} = -\theta_{\varepsilon}(\varepsilon_{ft} - 1)dt + \sigma_{\varepsilon}\sqrt{\varepsilon_{ft}} \, dB_{ft}, \tag{16}$$

$$du_{jt} = -\theta_u (u_{jt} - 1)dt + \sigma_u \sqrt{u_{jt}} \, dB_{jt}, \tag{17}$$

$$dx_t = \mu_x x_t dt + \sigma_x x_t dB_{xt}, \tag{18}$$

where  $dB_{ft}$ ,  $dB_{jt}$ , and  $dB_{xt}$  are increments of independent standard Brownian motions. At each time t, firm f acquires new projects according to a firm-specific Poisson process with a timevarying arrival rate given by  $\lambda_{ft} = \lambda_f \cdot \tilde{\lambda}_{ft}$ .<sup>14</sup> The constant  $\lambda_f$  captures the firm-specific long-run arrival rate of new projects and  $\tilde{\lambda}_{ft}$  follows a two-state continuous time Markov-chain with states  $\lambda_H > \lambda_L$ . Therefore, the intensity of project arrival is equal to  $\lambda_{ft} = \lambda_f \cdot \lambda_H$  in the high growth state and  $\lambda_{ft} = \lambda_f \cdot \lambda_L$  in the low growth state. The transition probabilities between time t and

<sup>&</sup>lt;sup>14</sup>At time t, the probability that firm f receives n project by time t + 1 is given by  $e^{-\lambda_{ft}} \frac{\lambda_{ft}^n}{n!}$ . The average number of projects received between t and t + 1 is  $\sum_{n=0}^{\infty} n e^{-\lambda_{ft}} \frac{\lambda_{ft}^n}{n!} = \lambda_{ft}$ . Hence,  $\lambda_{ft} dt$  represents the average number of projects received in the time interval dt.

t + dt into high-growth and low-growth states are  $\mu_H dt$  and  $\mu_L dt$ , respectively. Without loss of generality,  $\mathbb{E}[\tilde{\lambda}_{ft}] = 1$ .

Upon arrival of a new project j, the firm makes a take-it-or-leave-it decision. If the firm takes the project, it chooses the associated size of capital  $K_j$  and pays the corresponding investment expenditure of

$$i(x_t, z_t, K_j) = \frac{x_t}{z_t} K_j, \tag{19}$$

which depends on productivity,  $x_t$ , size of the new capital,  $K_j$ , and on the embodied *IST shock*,  $z_t$ . A positive realization of  $z_t$  reduces the cost of new capital investment. The process for IST shocks  $z_t$  also follows a geometric Brownian motion

$$dz_t = \mu_z z_t dt + \sigma_z z_t dB_{zt},\tag{20}$$

with  $dB_{zt}$  a standard Brownian motion independent of  $dB_{ft}$ ,  $dB_{jt}$ , and  $dB_{xt}$ . When a firm invests in a project *j*, the project-specific productivity is set to its long-run value  $u_{jt} = 1$ .

The stochastic discount factor  $\pi_t$  is given by

$$\frac{d\pi_t}{\pi_t} = -rdt - \gamma_x dB_{xt} - \gamma_z dB_{zt},\tag{21}$$

where r is the constant risk-free rate, and  $\gamma_x$  and  $\gamma_z$  are the constant prices of risk for the aggregate shocks  $x_t$  and  $z_t$ , respectively.

KP show that the value of assets in place (VAP) and the present value of growth opportunities (PVGO) for firm f are given, respectively, by

$$VAP_{ft} = x_t \sum_{j \in \mathcal{J}_t^f} A(\varepsilon_{ft}, u_{jt}) K_j^{\alpha}, \qquad (22)$$

$$PVGO_{ft} = x_t z_t^{\frac{\alpha}{1-\alpha}} G(\varepsilon_{ft}, \lambda_{ft}), \qquad (23)$$

where  $A(\varepsilon_{ft}, u_{jt})$  and  $G(\varepsilon_{ft}, \lambda_{ft})$  are defined in equations (11) and (16) of KP. The firm value is the sum of the two components,

$$P_{ft} = \text{VAP}_{ft} + \text{PVGO}_{ft}.$$
(24)

Hence, the firm's return IST exposure is given by

$$\beta_{ft}^{z} = \frac{\partial \ln P_{ft}}{\partial \ln z_{t}} = \frac{\alpha}{1 - \alpha} \frac{\text{PVGO}_{ft}}{P_{ft}}.$$
(25)

A firm's return exposure to IST shock is therefore proportional to the relative fraction of growth opportunities in the firm's total value. Unfortunately, because the fraction of growth opportunities in the firm value is not directly observable, to apply the above framework empirically, it is important to find an operational way to measure a firm's IST exposure.

#### 4.1.1 Investment-based IST betas

In this section, we show that the theoretical IST beta derived in equation (25) can also be expressed as a ratio of a firm's expected future investment expenditures and its market valuation. To see this, note that, because the arrival rate of new projects is exogenous, firms' investment decision follows a simple intra-temporal NPV rule. That is, at each time t firm f maximizes the project j's NPV:

$$NPV_{jt} = v(\varepsilon_{ft}, 1, x_t, K_j) - i(x_t, z_t, K_j).$$
(26)

where

$$v(\varepsilon_{ft}, u_{jt}, x_t, K_j) = \mathbb{E}_t \left[ \int_t^\infty e^{-\delta(s-t)} \frac{\pi_s}{\pi_t} \varepsilon_{fs} u_{js} x_s K_j^\alpha ds \right] = A(\varepsilon_{ft}, u_{jt}) x_t K_j^\alpha.$$
(27)

The optimal capital choice that maximizes the NPV (26) is given by

$$K_j^* = (\alpha z_t A(\varepsilon_{ft}, 1))^{\frac{1}{1-\alpha}}.$$
(28)

The firms' capital expenditure is then determined by equations (19) and (28). The following proposition formalizes how a firm's IST beta depends on investment expenditures.

**Proposition 1.** Under the assumption of the structural model of Section 4.1, firm f's stock return IST beta is given by

$$\beta_{ft}^{z} = \frac{\mathbb{E}_t \left[ \int_t^\infty e^{-\eta(s-t)} I_{fs} ds \right]}{P_{ft}},\tag{29}$$

where  $\eta = r + \gamma_x \sigma_x + \frac{\alpha}{1-\alpha} \gamma_z \sigma_z$ ,  $I_{fs} = i(x_s, z_s, K_s^*) \lambda_{fs}$  is firm f's average investment expenditures at time s, and  $P_{ft}$  is firm f's market value at time t.

The expression for IST beta in (29) confirms the intuition underlying the construction of the IST beta derived in equation (3). A positive and persistent IST shock decreases the cost of all future investment expenditures. In response to such a shock, the firm value *increases* by an amount that is proportional to the present value of all future investment expenditures. Therefore a firm's return exposure to a unit IST shock, that is its IST beta, is the present value of its future investment expenditures scaled by its current market value. Under the assumption of this model, the present value in equation (3) specializes to the case in which investment expenditures are discounted at a rate that is constant for all maturities. Proposition 1 justifies the use of a constant discount rate in the construction of the investment-based IST beta in equations (6) and (7).

Note that, as shown in equation (19), firms' investment expenditures in the model are driven by two aggregate shocks: the IST shock  $z_t$  and the productivity shock  $x_t$ . Therefore, equation (29) confirms our claim in Section 2.1 that the construction of IST beta from investment expenditures in equation (3) is valid even if investment expenditures are driven by multiple shocks.

If firm-specific productivity and project arrival rate are deterministic, the investment-based IST beta in equation (29) further simplifies to a quantity that is proportional to the ratio of *current* investment expenditures and current market capitalization, as illustrated in the following corollary:

**Corollary 1.** Assume that: (i) the firm-specific productivity  $\varepsilon_{ft}$  and (ii) the project arriving rate  $\lambda_{ft}$  are constant over time. Then, firm f's stock return IST beta is given by

$$\beta_{ft}^z = \frac{I_{ft}/P_{ft}}{\rho},\tag{30}$$

where  $\rho = \eta - \mu_x - \frac{\alpha}{1-\alpha}\mu_z - \frac{\sigma_z^2}{2}\frac{\alpha(2\alpha-1)}{(1-\alpha)^2}$ .

In the special case considered in the Corollary, the cross-sectional variation is driven only by project-specific shocks  $(u_{jt})$  and the Poisson randomness in the project arrival and death. Equation (30) shows that, in this case, a firm's I/P ratio is a direct measure of its IST-beta. Corollary 1 justifies the use of the scaled I/P ratio as a measure investment-based IST beta in equation (7).

#### 4.1.2 Comparison with proxy-based IST betas

By assuming the existence of an investment good sector supplying the capital good to the consumption good sector, KP further show that the return spread between investment and consumption good firms,  $IMC_t = r_t^I - r_t^C$ , is a mimicking factor for the IST shock.<sup>15</sup> Specifically, firm f's return exposure to IMC is given by

$$\beta_{ft}^{\rm IMC} \equiv \frac{\operatorname{cov}_t(r_{ft}, r_t^I - r_t^C)}{\operatorname{var}_t(r_t^I - r_t^C)} = \frac{1}{\beta_{0t}} \frac{\operatorname{PVGO}_{ft}}{P_{ft}}, \quad \text{where} \quad \beta_{0t} \equiv \frac{\int_{\mathcal{F}} \operatorname{VAP}_{ft} df}{\int_{\mathcal{F}} P_{ft} df}.$$
 (31)

Equation (31) defines a *proxy-based* measure of IST beta that can be constructed from financial data. Comparing the expression of  $\beta_{ft}^z$  in (25) to that of  $\beta_{ft}^{\text{IMC}}$  in (31), we can write

$$\beta_{ft}^{\rm IMC} = \frac{1}{\beta_{0t}} \frac{1-\alpha}{\alpha} \beta_{ft}^z.$$
(32)

The above equality illustrates that, within the model of this section, up to a scaling factor, a firm's proxy-based IST beta ( $\beta_{ft}^{\text{IMC}}$ ) coincides with its investment-based IST beta ( $\beta_{ft}^{z}$ ).<sup>16</sup>

Another commonly used approach to construct an IST proxy is to rely on the change in the relative price of new capital equipment (see, e.g., Greenwood, Hercowitz, and Krusell (1997)). In the context of the structural model of this section, these proxies capture the cost of per-unit capital in consumption units, that is,  $p_t^I = x_t/z_t$  in equation (19). Because it is affected by both neutral  $(x_t)$  and investment specific  $(z_t)$  shocks, the change in the price of capital  $\Delta p_t^I$ , cannot be uniquely linked to IST shocks  $z_t$ . One possible remedy for this measurement problem is to adjust the capital good price for the effect of productivity shocks  $x_t$ . This involves the construction of a "quality-adjusted" capital good price. For example, one can adjust the raw capital good price  $p_t^I = x_t/z_t$  by dividing it by the "quality" of the capital good, approximated by  $x_t$  and then obtain an adjusted capital good price  $p_{adj}^I = 1/z_t$ . The quantity  $Ishock \equiv -\ln p_{adj}^I$  can then be taken to be a proxy for the IST shock and, in turn, the IST beta can be estimated from the

<sup>&</sup>lt;sup>15</sup>The idea of using IMC as a measure of IST shocks is originally developed in Papanikolaou (2011).

<sup>&</sup>lt;sup>16</sup>The two are theoretically equivalent, conditional on the realization of the IST shock  $z_t$ . To see this, note that, from equations (22)–(24), the term  $\beta_{0t}$  in equation (31) depends on the aggregate IST shock  $z_t$ , but not on the neutral productivity shock  $x_t$ .

return exposure to the *Ishock*, that is,

$$\beta_{ft}^{Ishock} = \frac{\partial \ln P_{ft}}{\partial Ishock_t} = \frac{\partial \ln P_{ft}}{\partial \ln z_t} = \beta_{ft}^z.$$
(33)

The above equality indicates that, within the model of this section, a firm's return exposure to the quality-adjusted relative price of capital goods ( $\beta_{ft}^{Ishock}$ ) also coincides with its investment-based IST beta ( $\beta_{ft}^{z}$ ).

However, the above equivalence between the proxy-based and investment-based IST betas depends on the model specification. For example, a crucial condition for the IMC spread to be a measure of IST shocks is that investment- and consumption-good producers have the same exposure to the neutral productivity shock  $x_t$ . It is possible to show that, absent this condition, IMC is *not* a factor-mimicking portfolio for IST shocks.<sup>17</sup> Similarly, in order to measure precisely IST shocks, the quality adjustment on the relative price of capital goods is also model-dependent.

In summary, even though proxy-based and investment-based IST betas are theoretically equivalent within the selected structural model, the two approaches may generate different IST betas if either the model is misspecified or IST shocks are measured with error. In the next section we rely on simulation analysis to understand our earlier empirical findings as well as the economic mechanisms of the model.

## 4.2 Simulation analysis

In this section we rely on the structural model of Section 4.1 to understand the economic mechanisms underlying the observed empirical patterns of the I/K and I/P ratios documented in Table 1 (Section 4.2.1) and the investment-based and proxy-based betas documented in Table 2 (Section 4.2.2). In Section 4.2.3, we analyze firms' expected returns.

To understand the model's main economic mechanisms, we simulate it under two sets of parameters, reported in Table 3. The first set of parameters are those used by KP and reported in the column labeled "Baseline."<sup>18</sup> The second alternative set of parameters is reported in

<sup>&</sup>lt;sup>17</sup>A similar argument applies to an alternative proxy of IST shocks constructed from the the growth rate difference between total investment and consumption.

<sup>&</sup>lt;sup>18</sup>The only difference from the parameters used by KP is the distribution of mean project arrival rate  $\lambda_f$  which we take to be uniformly distributed between  $[\underline{\lambda}, \overline{\lambda}] = [5, 25]$ , as in Kogan and Papanikolaou (2013). Using the non-uniformly distributed  $\lambda_f$  as in Kogan and Papanikolaou (2014) gives the same results.

the column labeled "Alternative." Compared to the baseline parameters, in the alternative parameterization: (i) there are no firm-specific productivity shocks, that is  $\sigma_{\varepsilon} = 0$ ; (ii) the project-specific shock  $u_j$  is more persistent, that is  $\theta_u$  is smaller; and (iii) the difference in project arrivals between high and low growth states is less pronounced, that is the ratio  $\lambda_H/\lambda_L$  is smaller.<sup>19</sup> We emphasize that we choose the alternative parameterization to illustrate qualitative features of the model. Such a model may not necessarily be the true description of the cross section of firms exposed to investment shocks. A full calibration exercise is outside the scope of this paper.

We simulate the model at a weekly frequency and time-aggregate the results to obtain annual observations. We construct 1,000 samples of 2,500 firms over a period of 100 years and drop the first half of each sample to remove dependence on initial values. We report the median across samples of each variable of interest.

## 4.2.1 I/K vs. I/P ratios

Figure 5 reports I/K ratios and I/P ratios under the baseline (Panel A) and alternative parameterization (Panel B). The figure shows that under both parameterization, investment rates are decreasing in book-to-market, indicating that, consistent with the empirical findings in Panel A of Table 2, value firms have lower investment rates than growth firms. However, the patterns of I/P ratios across book-to-market portfolios are different under the two parameterizations. Specifically, in contrast with the findings in Panel B of Table 2, the model-implied I/Pratio under the baseline parameters is hump-shaped with value firms exhibiting a lower ratio than growth firms. I/P ratios are instead increasing in book-to-market under the alternative parameterization, consistent with the empirical findings reported in Panel B of Table 2.

Panel C and D of Figure 5 show a sharp contrast in the pattern of IST betas under the baseline and alternative parameterizations. Value firms have low IST betas than growth firms in the baseline parameterization while the opposite is true under the alternative parameter.

Note that, under the alternative parameterization, condition (i) of Corollary 1—no timevariation in firm-specific shock—is satisfied, while condition (ii)—no time-variation in project arrival rate—is violated. However, the time-variation in arrival rate is less extreme than under

<sup>&</sup>lt;sup>19</sup>The parameter choice in (i) and (ii) is the same as that of Kogan and Papanikolaou (2013).

the baseline parameterization. This explains why the I/P ratio and the IST-beta are both increasing in book-to market under the alternative parameters.

To understand the discrepancy in IST betas across parameterization, it is useful to analyze the connection between a firm's B/M and the value of its assets-in-place and growth options. To this purpose, let us rewrite the firm's book-to-market ratio as follows,

$$(B/M)_{ft} = \frac{K_{ft}}{P_{ft}} = \frac{K_{ft}}{VAP_{ft}} \times \left(1 - \frac{PVGO_{ft}}{P_{ft}}\right).$$
(34)

The above decomposition shows that if, in the cross-section, the B/M ratio is driven mainly by growth opportunities, that is, if  $K_{ft}/VAP_{ft}$  is roughly constant in (34), high B/M is associated with low  $PVGO_{ft}/P_{ft}$ . In contrast, if the B/M ratio in the cross section is driven mainly by the profitability of existing assets (captured by  $K_{ft}/VAP_{ft}$ ) with roughly constant growth opportunities ( $PVGO_{ft}$ ), high B/M is associated with low market value ( $P_{ft}$ ) and therefore high  $PVGO_{ft}/P_{ft}$ . Recall that the fraction of growth opportunities in firm value,  $PVGO_{ft}/P_{ft}$ , is proportional to the IST exposure (see equation (25)). Therefore, different economic mechanisms that drive the profitability of existing assets and growth opportunities would imply different patterns of investment shock betas across B/M portfolios.

The alternative parameterization reported in Table 3 is designed to capture a case in which the variation in B/M ratios is primarily driven by firm profitability instead of growth opportunities. Note, in fact, that in the alternative parameterization we differ from the baseline parameterization along three dimensions. First, we turn off the firm-specific productivity shock  $\varepsilon_f$  by setting  $\sigma_{\epsilon} = 0$ . This reduces one source of shock to growth opportunities and lower their importance in determining cross sectional variation across firms. Second, we reduce the speed of mean-reversion  $\theta_u$  of the project-specific shock  $u_j$  from 0.5 to 0.03. This increases the importance of the project-specific shock within the model. According to the decomposition (34), the more relevant the project-specific shock is, the more important is the role of the profitability of existing asset in determining the cross-sectional variation of B/M ratios. As discussed above, this implies a *positive* correlation between the B/M ratio and  $PVGO_{ft}/P_{ft}$ . Finally, we choose a smaller value for the ratio  $\lambda_H/\lambda_L$ . This ratio represents how many more projects a firm receives when it is in its high growth state relative to its low growth state. A smaller ratio of  $\lambda_H/\lambda_L$  reduces the Poisson randomness in terms of number of arriving projects. The Poisson randomness generates a mechanical size effect: a firm receiving more projects has more assets in place and hence a lower fraction of growth opportunities in firm value. Because a firm that receives more projects than expected has roughly the same  $K_{ft}/VAP_{ft}$  as one that receives less projects than expected, from decomposition (34) we have that a high  $\lambda_H/\lambda_L$  induces a negative relationship between B/M ratios and  $PVGO_{ft}/P_{ft}$ , or IST betas. By choosing a lower ratio  $\lambda_H/\lambda_L$  we reduce the randomness in project arrival and, consequently reduce the negative correlation between B/M and IST beta. These three mechanisms, combined together, help to generate a positive relationship between B/M ratio and the portfolio's IST exposure, thus explaining why the two parameterization generate opposite patterns in the IST betas as illustrated in Panels C and D of Figure 5.

### 4.2.2 Investment-based vs. proxy-based IST betas

In Section 3, we document that, in the data, the investment-based and proxy-based IST beta have different patterns across book-to-market portfolio. This finding is puzzling, considering the fact that, theoretically, they are equivalent in the model of Section 4.1 (see equations (32) and (33)). An important difference between the two types of beta is that while the construction of investment-based IST betas relies on fewer structural assumptions (see Section 2.1), proxy-based betas, such as those built from the IMC spread, rely on model-specific structural assumptions (see Section 4.1.2).

In this section, we assess the robustness of proxy-based IST betas to measurement error in the proxies. Specifically, we assume that the agent cannot observe directly the IST shock  $z_t$  but instead a version that is contaminated by other shocks. For simplicity, we assume that the agent observes

$$\frac{d\hat{z}_t}{\hat{z}_t} = \frac{dz_t}{z_t} + \xi_z \times r_t^m,\tag{35}$$

where  $z_t$  is the true IST shock in equation (20),  $r_t^m$  is the return on the market portfolio, and  $\xi_z$  is a constant capturing the severity of the measurement error. This distortion is meant to capture the fact that  $z_t$  is measured with error. For example,  $\hat{z}_t$  can represent the observed series like the quality-adjusted price of equipment *Ishock*, discussed in Section 3.2. In order for the noise to have an impact on the return betas, it has to be related to aggregate risk factors, and therefore the market return  $r_t^m$  is a natural and parsimonious choice for a "contaminating"

noise in (35). We follow a similar logic to construct a noisy IMC proxy and assume that the agent observes a distorted IMC spread,  $\widehat{IMC}$ , contaminated by the market return, that is,

$$\widehat{IMC} = IMC + \xi_{\rm IMC} \times r_t^m.$$
(36)

Panels A and B of Figure 6 report the "true" IST beta from the model  $\beta^z$ , as well as the investment-based beta,  $\beta_{20}^{IST}$ , and the IMC beta  $\beta^{IMC}$ . The left panel refers to the baseline parameterization in Table 3, while the right panel refers to the alternative parameterization. Both the investment-based and proxy based betas track the true beta patterns across B/M portfolios well. Note, however, that in the alternative parameterization, the exposure of value firms to IST shocks is *higher* than that of growth firms. This is a reflection of the different I/P patterns discussed in the previous section. Panels C and D report the noisy proxy-based betas. As we can see, when proxies are measured with error, the patterns of these betas can be the *opposite* of those in the true betas. Because the investment based beta is not affected by such measurement error, it will still provide the correct patterns across book-to-market portfolios.

In summary, while the investment-based IST beta introduced in this paper relies on a parsimonious set of assumptions, the existing proxy-based IST betas rely on the structural assumptions of theoretical models. As we show in this section, the observed opposite patterns of IST betas estimated from the two approaches may be related to the measurement error (or missing factors) in the IST proxies. Of course, the investment-based approach may also be sensitive to the way we analyze and measure investment data, even though as shown in Section 3.3, our main empirical findings survive a broad set of robustness tests.

#### 4.2.3 Expected returns

In the partial equilibrium model of Section 4.1, the prices of risk for both the IST and TFP shocks are exogenously specified in equation (21). The IST risk premium (per unit IST beta) for both sets of parameters used in our simulations is negative and equal to  $\lambda_z = \gamma_z \times \sigma_z = -0.35 \times 0.035 = -1.23\%$ . Because the return exposure to TFP shocks ( $\beta^x$ ) is the same across firms, the cross-sectional return difference is solely driven by the different exposure to IST shocks ( $\beta^z$ ).

Figure 7 reports the return patterns across ten book-to-market portfolios from our simulations. In the baseline parameterization (Panel A), the return is increasing with book-to-market, reflecting the fact that the chosen IST risk premium is negative and the IST beta is decreasing in book-to-market (see Panel C of Figure 5). In the alternative parameterization (Panel B), the return is instead decreasing in book-to-market because the IST beta is increasing in book-tomarket (see Panel D of Figure 5).

# 5 Discussion

In this section, we discuss the key implications of our empirical and simulation analyses of the previous two sections. We first highlight the important role of investment data in studying how IST shocks impact asset prices. We then discuss the potential implication of our analysis for the sign of the IST risk premium.

## 5.1 The importance of investment data

The analysis of the previous sections highlights several reasons for which investment data can play an important role in our understanding of the effect of IST shocks on asset prices.

First, investment data provide an independent estimate of IST betas. In our investmentbased approach, we rely on investment data to construct portfolio-level exposure to IST shocks. In particular, we used either the ex-post realized investment or the ex-ante expected future investment as the key ingredient to estimate firms' IST exposures. More important, these estimates are obtained without the need to rely on empirical proxies for IST shocks. As we show in Section 3, our investment-based IST betas increase monotonically with book-to-market ratio, in contrast to the pattern estimated from IST proxies. Hence, at the very least, the use of investment data enriches the set of empirical facts one needs to confront when analyzing the pricing impact of IST shocks.

To corroborate the empirical findings of Section 3 on book-to-market portfolios, we also implement our investment-based approach to estimate IST betas on investment rate (I/K)-sorted portfolios.<sup>20</sup> In untabulated results we find that: (i) high investment firms (high I/K portfolios)

<sup>&</sup>lt;sup>20</sup>We thank an anonymous referee for this suggestion.

have lower I/P ratios than low investment firms (low I/K portfolios); (ii) high investment firms have lower investment-based IST betas than low investment firms; but, in contrast, (iii) high investment firms have higher proxy-based IST betas than low investment firms. These findings provide another context in which I/K and I/P ratios, as well as investment-based and proxybased IST betas, differ in the cross-section. More important, the analysis on investment-sorted portfolios strengthen the main message of our analysis, that is, investment data play a key role in our understanding of the pricing effects of IST shocks.

Second, investment data provide a natural set of moments for realistic calibration of structural models. In particular, while existing studies emphasize the importance of investment rate (I/K) when studying the pricing impact of IST shocks, our approach highlights the importance of the investment to price ratio (I/P), which we show being directly related to IST betas. Therefore, we advocate that theoretical studies that use IST shocks be able to match the cross-sectional patterns of both investment rate and investment-to-price ratios.

Finally, the use of investment data may help to differentiate among competing economic mechanisms underlying the impact of IST shocks on the cross section of returns. As we show in Section 4.2.1, unlike the I/K ratio, the I/P ratio is very sensitive to the choice of economic mechanisms (represented by model parameters), suggesting that this ratio can serve as a good indicator to judge the quality of a model's calibration. For example, in the context of the theoretical model of Section 4.1, in order to match the pattern of I/P ratio across B/M portfolios as reported in Panel B of Table 1, one would have to choose the alternative parameterization over the baseline parameterization. The investment-to-price ratio I/P therefore contains important pricing information that should be taken into account in future theoretical work that addresses the pricing effect of investment shocks.

## 5.2 Implications for the IST risk premium

Note finally that different patterns of IST betas across book-to-market portfolios imply different sign of the implied IST risk-premium needed to explain the observed value premium. For example, in the baseline parameterization of the model in Section 4.1, a value of  $\lambda_z = -3.5\%$  and a IST beta spread of -2.3 between value and growth (Panel C of Figure 5), would imply a value premium of about 8%. In contrast, a value of  $\lambda_z = 11\%$  is needed in order to generate roughly the same magnitude of value premium in the alternative parameterization. A similar discussion also applies to the empirical patterns in the IST betas across book-to-market portfolios. As we report in Table 2, investment-based IST beta is increasing monotonically with book-to-market, while the proxy-based IST beta is either flat ( $\beta_{IMC}$ ) or hump-shaped ( $\beta_{Ishock}$ ) with lower beta for value firms. Therefore, if we were to rely on IST shocks to account for the entire magnitude of the value premium, we would need a *positive* IST risk-premium, when using investment-based IST betas, but a *negative* IST risk-premium, when using the proxy-based  $\beta_{Ishock}$ .

The existing literature provides general equilibrium theories linking investment shocks and returns. These theories differs in their implications for the sign of the IST risk premium. For example, Papanikolaou (2011) shows that the IST risk premium is *negative* if capital utilization is fixed and agents have preference for late resolution of uncertainty. In contrast, Garlappi and Song (2017b) show that the IST risk premium is *positive* if capital utilization is flexible and agents have preference for early resolution of uncertainty. Our empirical finding that value firms have higher investment-based IST betas than growth firms imply that, for IST shocks to contribute positively to the observed value premium, a positive IST risk premium is required.

Although we ignore general equilibrium effects in our analysis, the empirical finding that value firms have higher IST exposure than growth firms offers some guidance on the economic mechanism that links IST shocks and cross-sectional returns. In particular, given the theoretical link between investment-to-price ratios and firms' return exposure to IST shocks, the empirical evidence of Section 3 indicates that future theories connecting IST shocks to cross-sectional returns should aim at generating empirically consistent patterns of both I/P ratios and investment-based IST betas in the cross-section of firms. In this regard, we believe our partial equilibrium analysis in this paper can serve as an benchmark for future studies.

# 6 Conclusion

In this paper we provide a new methodology for estimating a firm's stock return sensitivity to capital-embodied technology shocks. Our methodology is based on the idea that a firm's investment contains useful information regarding its exposure to IST shocks. Empirically, we find that investment-based IST betas are higher for value stocks than for growth stocks, contradicting the findings in the existing studies based on IST proxies. To better understand these new empirical findings, we analyze in depth the economic mechanisms of a well-studied structural model of investment. We show that, within this model, our investment-based IST betas provide good estimates of the true IST betas, while proxy-based IST betas are vulnerable to measurement errors. More important, our analysis highlights the key role played by investment data when studying the possible economic mechanisms through which IST shocks affect asset prices in the cross section.

We acknowledge that our investment-based approach has potential limitations. First, we only focus on the partial equilibrium impact of IST shocks on new investment, and ignore the general equilibrium effect on the value of assets in place. Second, the investment data we use may also contain measurement errors or biases. Notwithstanding these limitations, we view our contribution as providing an alternative, theory-based, approach to estimate firms' return exposures to IST shocks. Furthermore, we think that the empirical facts documented in this paper enrich our perspective on the pricing of investment shocks.

In light of the discrepancy between investment-based and proxy-based inference, exploring alternative measures of IST shocks to those available in the existing literature is of first-order importance for gaining a deeper understanding of their effect on returns. We believe that the new methodology proposed in this paper represents a useful benchmark for assessing the validity of alternative measures of capital-embodied technical change and their impact on asset prices.

# A Proofs

# **Proof of Proposition 1**

Using the optimal investment scale  $K^*$  from (28), the investment cost in (19) is

$$i(x_t, z_t, K_t^*) = x_t z_t^{\frac{\alpha}{1-\alpha}} (\alpha A(\varepsilon_{ft}, 1))^{\frac{1}{1-\alpha}},$$
(A.1)

and hence, from (26) we have

$$NPV_t^* = \frac{1-\alpha}{\alpha} i(x_t, z_t, K_t^*).$$
(A.2)

Direct calculations yield that the present value at time t of firm f's growth opportunities,  $PVGO_{ft}$ , is given by:

$$\begin{aligned} \operatorname{PVGO}_{ft} &= \mathbb{E}_t \left[ \int_t^\infty \frac{\pi_s}{\pi_t} \operatorname{NPV}_s^* \lambda_{fs} ds \right] \end{aligned} \tag{A.3} \\ &= \frac{1-\alpha}{\alpha} \mathbb{E}_t \left[ \int_t^\infty \frac{\pi_s}{\pi_t} x_s z_s^{\frac{\alpha}{1-\alpha}} (\alpha A(\varepsilon_{fs}, 1))^{\frac{1}{1-\alpha}} \lambda_{fs} ds \right] \\ &= \frac{1-\alpha}{\alpha} \int_t^\infty \mathbb{E}_t^{x,z} \left[ \frac{\pi_s}{\pi_t} x_s z_s^{\frac{\alpha}{1-\alpha}} \right] \mathbb{E}_t^{\varepsilon,\lambda} \left[ (\alpha A(\varepsilon_{fs}, 1))^{\frac{1}{1-\alpha}} \lambda_{fs} \right] ds \\ &= \frac{1-\alpha}{\alpha} \int_t^\infty e^{-(r+\gamma_x \sigma_x + \frac{\alpha}{1-\alpha} \gamma_z \sigma_z)(s-t)} \mathbb{E}_t^{x,z} \left[ x_s z_s^{\frac{\alpha}{1-\alpha}} \right] \mathbb{E}_t^{\varepsilon,\lambda} \left[ (\alpha A(\varepsilon_{fs}, 1))^{\frac{1}{1-\alpha}} \lambda_{fs} \right] ds \\ &= \frac{1-\alpha}{\alpha} \mathbb{E}_t \left[ \int_t^\infty e^{-\eta(s-t)} i(x_s, z_s, K_s^*) \lambda_{fs} ds \right] \\ &= \frac{1-\alpha}{\alpha} \mathbb{E}_t \left[ \int_t^\infty e^{-\eta(s-t)} I_{fs} ds \right], \end{aligned}$$

where the first equality is the definition of  $PVGO_{ft}$ ; the second equality follows from (A.2) and (A.1); the third equality uses the fact that the processes  $x_t$  and  $z_t$  are independent of  $\lambda_{ft}$ and  $\varepsilon_{ft}$  thus allowing to express the expectation  $\mathbb{E}_t$  as the product of the expectation under the measure governing the dynamics of  $x_t$  and  $z_t$ ,  $\mathbb{E}_t^{x,z}$ , and the expectation under the measure governing the dynamics of  $\varepsilon_{ft}$  and  $\lambda_t$ ,  $\mathbb{E}_t^{\varepsilon,\lambda}$ ;<sup>21</sup> the fourth equality exploits the fact that  $x_t$  and  $z_t$  are geometric Brownian motions, defined in (18) and (20); the fifth equality follows from the independence of the stochastic processes  $x_t$ ,  $z_t$ ,  $\varepsilon_{ft}$  and  $\lambda_{ft}$ , and uses  $\eta \equiv r + \gamma_x \sigma_x + \frac{\alpha}{1-\alpha} \gamma_z \sigma_z$ , and

<sup>&</sup>lt;sup>21</sup>If  $PVGO_{ft} < \infty$ , by Fubini's Theorem, we can interchange expectation and integration.

the definition of optimal investment in (A.1); and, the last equality defines average investment at time s,  $I_{fs} \equiv i(x_s, z_s, K_s^*)\lambda_{fs}$ .

Using (A.4) in the definition of firm f's IST beta (25), we then obtain equation (29) in Proposition 1.

# Proof of Corollary 1

If  $\varepsilon_{ft}$  and  $\lambda_{ft}$  are constant, then the expression of PVGO<sub>ft</sub> in (A.4) simplifies further as follows,

$$\begin{aligned} \operatorname{PVGO}_{ft} &= \frac{1-\alpha}{\alpha} \left[ (\alpha A(\varepsilon_f, 1))^{\frac{1}{1-\alpha}} \lambda_f \right] \int_t^\infty e^{\left(-r - \gamma_x \sigma_x - \frac{\alpha}{1-\alpha} \gamma_z \sigma_z\right)(s-t)} \mathbb{E}_t^{x,z} \left[ x_s z_s^{\frac{\alpha}{1-\alpha}} \right] ds \quad (A.4) \\ &= \frac{1-\alpha}{\alpha} \left[ (\alpha A(\varepsilon_f, 1))^{\frac{1}{1-\alpha}} x_t z_t^{\frac{\alpha}{1-\alpha}} \lambda_f \right] \mathbb{E}_t \left[ \int_t^\infty e^{-\rho(s-t)} ds \right] \\ &= \frac{1-\alpha}{\alpha} \frac{i(x_t, z_t, K_t^*) \lambda_f}{\rho} \\ &= \frac{1-\alpha}{\alpha} \frac{I_{ft}}{\rho}, \end{aligned}$$

where  $\rho = \eta - \mu_x - \frac{\alpha}{1-\alpha}\mu_z - \frac{\sigma_z^2}{2}\frac{\alpha(2\alpha-1)}{(1-\alpha)^2}$ . Using (A.5) in the definition of firm *f*'s IST beta (25), we then obtain equation (30) in Corollary 1.

# **B** Data details

In this appendix, we provide further details on the construction of portfolios and portfolio-level variables.

**Book-to-market portfolios.** To calculate a firm's book-to-market ratio, we use firm's book equity for the fiscal year end (as in Fama and French (2008a,b)) and its market capitalization (price×share outstanding) at the end of December. If Compustat book equity is missing, we use the historical book equity as in Davis, Fama, and French (2000), available from Ken French's website. We sort stocks into ten portfolios according to firms' book-to-market ratio at the end of each year and keep track of the portfolio for subsequent years. We report the value-weighted returns for each book-to-market decile.

**Portfolio-level investment ratios.** We construct the portfolio-level I/K and I/P ratios from firm-level data. Because, after portfolio formation we track the firms in each portfolio for many years, some firms will naturally exit the sample. To deal with missing observations, we follow two different approaches. In the benchmark analysis, we only use firms with non-missing capital expenditure data to construct the portfolio-level ratios as,

$$\left(\frac{I_{t+s-1}}{K_{t+s-2}}\right)_{\text{Portfolio}_i} = \frac{\sum_{f \in \text{Portfolio}_i \& \text{non-missing } I_{f,t+s-1}}{\sum_{f \in \text{Portfolio}_i \& \text{non-missing } I_{f,t+s-1}} K_{f,t+s-2}, \ i = 1, \dots, 10; \ s = 1, \dots, T.(B.1)$$

$$\left(\frac{I_{t+s-1}}{P_{t-1}}\right)_{\text{Portfolio}_i} = \frac{\sum_{f \in \text{Portfolio}_i \& \text{non-missing } I_{f,t+s-1}}{\sum_{f \in \text{Portfolio}_i \& \text{non-missing } I_{f,t+s-1}} P_{f,t-1}}, \ i = 1, \dots, 10; \ s = 1, \dots, T.(B.2)$$

The implied assumption in this approach is that the portfolio I/K and I/P ratios are computed as if firms with missing investment data have the same average I/K or I/P ratios as that of firms with non-missing data in the same portfolio. Alternatively, in the robustness analysis, we assume that firms with missing investment data make zero investment, and construct the portfolio-level ratios as,

$$\left(\frac{I_{t+s-1}}{K_{t+s-2}}\right)_{\text{Portfolio}_i} = \frac{\sum_{f \in \text{Portfolio}_i \& \text{non-missing } I_{f,t+s-1}}{\sum_{f \in \text{Portfolio}_i K_{f,t+s-2}}, \quad i = 1, \dots, 10; \ s = 1, \dots, T(B.3)$$
$$\left(\frac{I_{t+s-1}}{P_{t-1}}\right)_{\text{Portfolio}_i} = \frac{\sum_{f \in \text{Portfolio}_i \& \text{non-missing } I_{f,t+s-1}}{\sum_{f \in \text{Portfolio}_i P_{f,t-1}}}, \quad i = 1, \dots, 10; \ s = 1, \dots, T(B.4)$$

Since firms with missing investment data may incur in non-negative investment expenditure, this approach puts a lower bound on the investment ratios for each portfolio.



Figure 2: Investment-based IST betas of B/M portfolios: different parameters

The figure reports the investment-based IST betas of book-to-market portfolios estimated from data on U.S. common stocks over the period 1963–2016.  $\beta_1^{\text{IST}}$  is constructed from equation (7),  $\beta_{15}^{\text{IST}}$  and  $\beta_{20}^{\text{IST}}$  are constructed from equation (6). Panel A reports the benchmark case in which the discount rates  $\rho = 0.04, \eta = 0.12$ . Panel B reports the results from discounting the investment by the portfolio realized average returns according to equations (11) and (12). Panel C reports the results in which we adjust investment expenditures according to equation (13) with  $\theta = 0.1$ . Panel D reports the results obtained by adjust both the investment and the discount rates.



Figure 3: Investment-based IST betas of B/M portfolios: different methodology

The figure reports the investment-based IST betas of book-to-market portfolios estimated from data on U.S. common stocks over the period 1963–2016.  $\beta_1^{\text{IST}}$  is constructed from equation (7),  $\beta_{15}^{\text{IST}}$  and  $\beta_{20}^{\text{IST}}$  are constructed from equation (6). Panel A reports the benchmark case in which the delisted firms are assumed to have the same investment-to-price ratio as the average firm in the same portfolio. Panel B reports the results in which the delisted firms are assumed to have zero investment expenditure after delisting. Panel C reports the results in which the betas are computed by using the expected investment estimated from the cross-sectional regressions in equation (14). Panel D reports the results that combine both the expected investment and no-adjustment for delisting. In all cases, the discount rates  $\rho = 0.04, \eta = 0.12$ .



### Figure 4: Investment-based IST betas of B/M portfolios: different samples

The figure reports the investment-based IST betas of book-to-market portfolios estimated from data on U.S. common stocks over the period 1963–2016.  $\beta_1^{\text{IST}}$  is constructed from equation (7),  $\beta_{15}^{\text{IST}}$  and  $\beta_{20}^{\text{IST}}$  are constructed from equation (6). Panel A reports the benchmark case using non-financial firms. Panel B reports the results using non-financial and consumption firms. Panel C reports the results using firms that have fiscal year-end of December. Panel D reports the results obtained from non-financial and consumption firms which have December fiscal year-end. In all cases, the discount rates  $\rho = 0.04$ ,  $\eta = 0.12$ .



Baseline parameters

# Alternative parameters

Figure 5: Model implications under alternative parameterizations

The figure reports investment rates (I/K), investment-to-price (I/P) ratios, and IST betas for ten book-to-market portfolios obtained from two alternative parameterizations of the structural model of Section 2. The parameter values used in each simulations are reported in Table 3. We simulate 1,000 samples of 2,500 firms over a period of 100 years and drop the first half of each sample to remove dependence on initial values. The figure reports the median numbers across the 1,000 samples.



Alternative parameters



Figure 6: The effect of measurement error in IST proxies

The left panels refer to the baseline parameters in Table 3 while the right panels refer to the alternative parameterization. We simulate 1,000 samples of 2,500 firms over a period of 100 years and drop the first half of each sample to remove dependence on initial values. The figure reports the median numbers across the 1,000 samples. Panels A and B report "true" IST betas from the model.  $\beta^z$  is obtained by projecting simulated returns on the IST shock  $dz_t/z_t$ .  $\beta^{IMC}$  is obtained by projecting simulated returns on the IST shock  $dz_t/z_t$ .  $\beta^{IMC}$  is obtained by projecting simulated returns on the IST shock  $dz_t/z_t$ .  $\beta^{IMC}$  is obtained by projecting simulated returns on the simulated series of IMC.  $\beta_{20}^{IST}$  is the investment-based beta from equation (6) with T = 20. Panels C and D report noisy proxy betas.  $\tilde{\beta}^z$  is obtained by using the noisy proxy for z-shock in (35) with  $\xi_z = -0.4$  and  $\tilde{\beta}^{IMC}$  is obtained by using the noisy proxy for IMC in (36) with  $\xi_{IMC} = -0.9$ . Both  $\tilde{\beta}^z$  and  $\tilde{\beta}^{IMC}$  are estimated from bivariate regressions with the second factor being the noisy x-shock:  $d\hat{x}_t/\hat{x}_t = dx_t/x_t - 0.4 \times r_t^m$ .



Figure 7: Portfolio returns under alternative parameterizations

The figure reports returns for ten book-to-market portfolios obtained from two alternative parameterizations of the structural model of Section 4. The parameter values used in each simulations are reported in Table 3. We simulate 1,000 samples of 2,500 firms over a period of 100 years and drop the first half of each sample to remove dependence on initial values. The figure reports the median numbers across the 1,000 samples of the average returns for each portfolio.

#### Table 1: Investment patterns of book-to-market portfolios

This table reports the investment patterns of book-to-market portfolios over a window of 15 years. The sample covers the period of 1963–2016. After forming portfolios at the end of each year (t), we track each portfolio for the next 15 years (the last portfolio-formation year is 2001 because we need 15 years of post-formation investment data). Panel A reports the investment-to-capital ratio (I/K), which is defined as the capital expenditure divided by the lagged capital  $(I_{t+s}/K_{t+s-1})$ . Panel B reports the investment-to-price ratio (I/P), which is defined as the present value of capital expenditure divided by the current equity value  $(PV(I_{t+s})/P_t)$ . The investment  $I_{t+s}$  is the capital expenditure during year t + s, the lagged capital  $K_{t+s-1}$  is the property, plant and equipment-total (net) at the fiscal year end in t + s - 1, and  $P_t$  is the market capitalization at the end of formation year t. The PV $(I_{t+s})$  is the present value at year t of investment expenditure at year t + s, by using a constant discount rate of 12% per year. All the reported values are the time series averages over all the portfolio-formation years (1962–2001).

		post-formation year $(s)$													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Panel A: Investment-to-capital ratio: $I/K$															
Low $B/M$	0.31	0.30	0.29	0.27	0.27	0.26	0.26	0.25	0.25	0.24	0.24	0.24	0.24	0.24	0.23
2	0.26	0.25	0.24	0.24	0.23	0.23	0.22	0.22	0.22	0.22	0.21	0.21	0.20	0.20	0.20
3	0.23	0.23	0.23	0.22	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.20	0.20	0.20	0.19
4	0.21	0.21	0.21	0.21	0.20	0.20	0.20	0.19	0.19	0.19	0.19	0.19	0.19	0.18	0.18
5	0.20	0.20	0.20	0.19	0.19	0.18	0.18	0.18	0.18	0.18	0.18	0.17	0.17	0.17	0.17
6	0.18	0.18	0.18	0.18	0.17	0.17	0.17	0.17	0.17	0.16	0.16	0.16	0.16	0.17	0.16
7	0.17	0.17	0.16	0.16	0.16	0.16	0.15	0.15	0.15	0.15	0.15	0.15	0.16	0.15	0.16
8	0.15	0.15	0.15	0.15	0.15	0.15	0.14	0.14	0.15	0.15	0.15	0.15	0.15	0.15	0.15
9	0.14	0.14	0.14	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.16	0.16
High $B/M$	0.13	0.13	0.14	0.15	0.15	0.15	0.16	0.16	0.16	0.16	0.16	0.16	0.17	0.16	0.16
HML	-0.18	-0.17	-0.15	-0.13	-0.11	-0.11	-0.10	-0.09	-0.09	-0.09	-0.08	-0.08	-0.07	-0.07	-0.07
Panel B: Investment-to-price ratio: $I/P$															
Low $B/M$	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
2	0.07	0.07	0.07	0.07	0.07	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05
3	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.06	0.06
4	0.11	0.11	0.11	0.11	0.11	0.10	0.10	0.10	0.09	0.09	0.09	0.08	0.08	0.08	0.07
5	0.14	0.13	0.13	0.13	0.12	0.12	0.12	0.11	0.11	0.10	0.10	0.09	0.09	0.08	0.08
6	0.15	0.15	0.15	0.15	0.14	0.13	0.13	0.12	0.12	0.11	0.11	0.10	0.10	0.10	0.09
7	0.17	0.17	0.16	0.16	0.15	0.15	0.14	0.13	0.13	0.12	0.12	0.11	0.11	0.10	0.10
8	0.19	0.18	0.18	0.17	0.16	0.16	0.15	0.13	0.13	0.13	0.12	0.12	0.12	0.11	0.11
9	0.21	0.20	0.19	0.19	0.18	0.17	0.17	0.16	0.15	0.15	0.14	0.14	0.13	0.13	0.13
High $B/M$	0.29	0.26	0.26	0.26	0.26	0.25	0.24	0.23	0.22	0.21	0.21	0.21	0.20	0.19	0.18
HML	0.25	0.22	0.22	0.22	0.22	0.21	0.20	0.19	0.18	0.18	0.17	0.17	0.17	0.15	0.15

#### Table 2: Returns and IST betas of book-to-market portfolios

This table reports the returns and IST exposures of book-to-market portfolios. Panel A reports the portfolio return in excess of risk-free rate. Panel B reports the investment-based IST betas and panel C reports the proxy-based IST betas. The return is calculated using the data one year after the portfolio formation. We use capital expenditure data to estimate the IST exposures  $\beta_1^{\text{IST}}$  and  $\beta_{15}^{\text{IST}}$  according to equations (7) and (6) with  $\rho = 0.04$  and  $\eta = 0.12$ . Investment-based betas are the average across years from 1963 to 2016 for  $\beta_1^{\text{IST}}$  and 1963 to 2002 for  $\beta_{15}^{\text{IST}}$  (note that the last  $\beta_{15}^{\text{IST}}$  in 2002 is constructed based on investment from 2002-2016). We use two proxies of IST shocks, *Ishock* and *IMC*, to estimate the proxy-based IST betas. We consider both one-factor and two-factor models, with the second factor to be either the market excess return (MKT) or the growth rate of total factor of productivity (TFP). The proxy-based beta estimates are based on the time series of 1963 to 2016. The *t*-statistics (in parentheses) are Newey-West adjusted with a lag length of 2 years.

		Low	2	3	4	5	6	7	8	9	High	HML
Panel A: Returns												
	Ret (%)	6.37 (2.41)	5.94 (2.75)	6.81 (3.21)	6.44 (3.19)	6.92 (3.06)	$8.86 \\ (4.88)$	9.37 (4.81)	9.68 (4.71)	10.22 (5.04)	14.63 (5.44)	$8.26 \\ (2.53)$
Panel B: Inv	estment-b	based IS7	f betas									
	$\beta_1^{\rm ist}$	1.09 (7.24)	1.72 (7.47)	2.29 (7.19)	2.76 (7.72)	3.39 (8.21)	3.84 (8.40)	4.34 (8.16)	4.76 (9.94)	5.36 (10.28)	7.23 (9.58)	6.14 (9.61)
	$\beta_{15}^{\rm ist}$	1.52 (8.13)	$2.16 \\ (7.66)$	2.82 (8.59)	$3.31 \\ (7.93)$	3.67 (11.39)	4.26 (9.58)	4.48 (9.62)	4.99 (7.42)	5.72 (7.90)	8.12 (6.63)	$6.60 \\ (5.87)$
Panel C: Pro	oxy-based	IST bet	as									
IST only:												
·	$\beta_{Ishock}$	-2.02 (-2.07)	-1.79 (-1.86)	-1.83 (-1.79)	-1.94 (-1.39)	-1.51 (-1.56)	-1.83 (-1.45)	-2.97 (-3.16)	-3.30 (-2.89)	-2.87 (-2.84)	-3.43 (-2.37)	-1.41 (-1.24)
	$\beta_{IMC}$	$\begin{array}{c} 0.43 \\ (1.81) \end{array}$	$\begin{array}{c} 0.32 \\ (1.60) \end{array}$	$\begin{array}{c} 0.38\\ (2.08) \end{array}$	$\begin{array}{c} 0.37 \\ (2.06) \end{array}$	0.34 (1.77)	$\begin{array}{c} 0.35 \\ (2.24) \end{array}$	$\begin{array}{c} 0.21 \\ (0.88) \end{array}$	$\begin{array}{c} 0.24 \\ (0.92) \end{array}$	$\begin{array}{c} 0.29 \\ (1.37) \end{array}$	$\begin{array}{c} 0.57 \\ (1.72) \end{array}$	$0.14 \\ (0.44)$
151+MK1:	$\beta_{Ishock}$	0.14 (0.32)	0.28 (1.38)	$0.13 \\ (0.61)$	-0.06 (-0.08)	$0.38 \\ (0.85)$	-0.16 (-0.24)	-1.31 (-2.62)	-1.55 (-2.20)	-1.17 (-2.14)	-1.24 (-1.36)	-1.38 $(-1.12)$
	$\beta_{IMC}$	$\begin{array}{c} 0.10 \\ (0.89) \end{array}$	$\begin{array}{c} 0.00 \\ (0.01) \end{array}$	$0.08 \\ (1.24)$	$0.08 \\ (0.69)$	$0.06 \\ (0.65)$	$\begin{array}{c} 0.09 \\ (0.90) \end{array}$	-0.08 (-0.51)	-0.07 (-0.42)	$0.00 \\ (0.00)$	$\begin{array}{c} 0.21 \\ (0.89) \end{array}$	$\begin{array}{c} 0.11 \\ (0.38) \end{array}$
151+1FP:	$\beta_{Ishock}$	-2.02 (-2.11)	-1.79 (-1.92)	-1.83 (-1.81)	-1.94 (-1.46)	-1.51 (-1.65)	-1.83 $(-1.51)$	-2.97 (-3.49)	-3.31 (-3.13)	-2.87 (-3.02)	-3.44 (-2.49)	-1.42 (-1.23)
	$\beta_{IMC}$	$\begin{array}{c} 0.42 \\ (1.80) \end{array}$	$\begin{array}{c} 0.31 \\ (1.55) \end{array}$	$\begin{array}{c} 0.38\\ (2.07) \end{array}$	$\begin{array}{c} 0.35 \\ (1.96) \end{array}$	$\begin{array}{c} 0.33 \\ (1.66) \end{array}$	$\begin{array}{c} 0.34 \\ (2.20) \end{array}$	$\begin{array}{c} 0.18 \\ (0.74) \end{array}$	$\begin{array}{c} 0.21 \\ (0.81) \end{array}$	$0.26 \\ (1.25)$	$\begin{array}{c} 0.53 \\ (1.56) \end{array}$	$\begin{array}{c} 0.11 \\ (0.32) \end{array}$

## Table 3: Parameter values

This table summarizes the two sets of alternative parameter values used in simulations of the model described in Section 4.

Parameter	Symbol	Values		
		Baseline	Alternative	
Technology, aggregate shocks				
Mean growth rate of the disembodied technology shock	$\mu_x$	0.005		
Volatility of the disembodied technology shock	$\sigma_x$	0.135		
Mean growth rate of the IST shock	$\mu_z$	0.002		
Volatility of the IST shock	$\sigma_z$	0.035		
Technology, idiosyncratic shocks				
Persistence of the firm-specific shock	$ heta_arepsilon$	0.350		
Volatility of the firm-specific shock	$\sigma_{arepsilon}$	0.200	0.000	
Persistence of the project-specific shock	$ heta_u$	0.500	0.030	
Volatility of the project-specific shock	$\sigma_u$	1.500		
Project arrival and depreciation				
Project depreciation rate	$\delta$	0.100		
Firm-specific long-run arrival rate–lower bound	$\underline{\lambda}$	5.000		
Firm-specific long-run arrival rate–upper bound	$\overline{\lambda}$	25.000		
Transition probability into high-growth state	$\mu_H$	0.075		
Transition probability into low-growth state	$\mu_L$	0.160		
Ratio of arrival rates in high vs. low growth states	$\lambda_H/\lambda_L$	6.400	2.000	
Stochastic discount factor				
Risk-free rate	r	0.030		
Price of risk of the disembodied shock	$\gamma_x$	0.690		
Price of risk of the IST shock	$\gamma_z$	-0.350		
Other				
Project-level return-to-scale parameter	$\alpha$	0.850		
Profit margin of the investment sector	$\phi$	0.070		

# References

- Berk, J. B., R. C. Green, and V. Naik, 1999, "Optimal Investment, Growth Options, and Security Returns," *Journal of Finance*, 54, 1553–1607.
- Carlson, M., A. Fisher, and R. Giammarino, 2004, "Corporate Investment and Asset Price Dynamics: Implications for the Cross-section of Returns," *Journal of Finance*, 59, 2577– 2603.
- Christiano, L. J., and J. D. M. Fisher, 2003, "Stock Market and Investment Goods Prices: Implications for Macroeconomics," NBER Working paper no.10031.
- Cochrane, J. H., 1991, "Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations," *Journal of Finance*, 46, 209–237.
- ——, 1996, "A Cross-Sectional Test of an Investment-Based Asset Pricing Model," Journal of Political Economy, 104, 572–621.
- Davis, J. L., E. F. Fama, and K. R. French, 2000, "Characteristics, Covariances and Average Returns: 1929-1997," *Journal of Finance*, 55, 389–406.
- Dissanayake, R., A. Watanabe, and M. Watanabe, 2017, "Investment Shocks and Asset Returns: International Evidence," University of Alberta School of Business Research Paper No. 2016-511.
- Fama, E. F., and K. R. French, 1993, "Common Risk Factors in the Returns on Stocks and Bonds," Journal of Financial Economics, 33, 3–56.
- ——, 2008a, "Average Returns, B/M, and Share Issues," Journal of Finance, 63, 2971–2995.
- ——, 2008b, "Dissecting Anomalies," Journal of Finance, 63, 1653–1678.
- Fisher, J. D. M., 2006, "The Dynamic Effects of Neutral and Investment-specific Technology Shocks," *Journal of Political Economy*, 114, 413–451.
- Garlappi, L., and Z. Song, 2017a, "Can Investment Shocks Explain the Cross Section of Equity Returns?," *Management Science*, 63, 3829–3848.
- —— , 2017b, "Capital Utilization, Market Power, and the Pricing of Investment Shocks," Journal of Financial Economics, 126, 447–470.
- Garleanu, N., L. Kogan, and S. Panageas, 2012, "Displacement Risk and Asset Returns," Journal of Financial Economics, 105, 491–510.
- Garleanu, N., S. Panageas, and J. Yu, 2012, "Technological Growth and Asset Pricing," Journal of Finance, 67, 1265–1292.
- Gomes, J., L. Kogan, and L. Zhang, 2003, "Equilibrium Cross Section of Returns," Journal of Political Economy, 111, 693–732.

- Gomes, J. F., L. Kogan, and M. Yogo, 2009, "Durability of Output and Expected Stock Returns," Journal of Political Economy, 117, 941–986.
- Greenwood, J., Z. Hercowitz, and P. Krusell, 1997, "Long-run Implications of Investment-specific Technological Change," American Economic Review, 87, 342–362.
- ——, 2000, "The Role of Investment-specific Technological Change in the Business Cycle," *European Economic Review*, 44, 91–115.
- Israelsen, R., 2010, "Investment Based Valuation and Mangerial Expectations," Working paper.
- Justiniano, A., G. E. Primiceri, and A. Tambalotti, 2010, "Investment Shocks and Business Cycles," *Journal of Monetary Economics*, 57, 132–145.
- Kogan, L., and D. Papanikolaou, 2013, "A Theory of Firm Characteristics and Stock Returns: The Role of Investment-Specific Shocks," *Review of Financial Studies*, 26, 2718–2759.
- —— , 2014, "Growth Opportunities, Technology Shocks, and Asset Prices," Journal of Finance, 69, 675–718.
- Li, J., 2018, "Explaining Momentum and Value Simultaneously," *Management Science*, 64(9), 4239–4260.
- Li, J., and H. Wang, 2017, "Expected Investment Growth and the Cross Section of Stock Returns," Working paper.
- Lin, X., and L. Zhang, 2013, "The Investment Manifesto," Journal of Monetary Economics, 60, 351–366.
- Liu, L. X., T. M. Whited, and L. Zhang, 2009, "Investment-based Expected Stock Returns," Journal of Political Economy, 117, 1105–1139.
- Papanikolaou, D., 2011, "Investment Shocks and Asset Prices," Journal of Political Economy, 119, 639–685.
- Solow, R. M., 1960, "Investment and Technical Progress," In K. J. Arrow, A. Karlin, and P. Suppes (Eds.), Mathematical Methods in the Social Sciences, pp. 89-104. Stanford, CA: Stanford University Press.
- Yang, F., 2013, "Investment Shocks and the Commodity Basis Spread," Journal of Financial Economics, 110, 164–184.
- Zhang, L., 2005, "The Value Premium," Journal of Finance, 60, 67–103.