Net Trade and Market Efficiency in Grossman and Stiglitz (1980)

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Abstract

In this paper, we correct part (b) of Theorem 6 of Grossman and Stiglitz (GS, 1980). We demonstrate that when the private signal tends to be perfect, the market converges to strong-form efficiency, and thus informed and uninformed traders have almost homogeneous beliefs about the stock payoff, but there is still significant net trade, rather than no trade as erroneously shown by GS. We further show that when the stock price becomes more informative, and thus traders' beliefs about the stock payoff become closer, the net trade may increase.

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1 Introduction

Grossman and Stiglitz (GS, 1980) develop a competitive equilibrium asset pricing model with asymmetric information between informed traders who acquire a private signal about the stock payoff at a cost and uninformed traders who extract a noisy version of the private signal freely from the stock price. Theorem 5 of GS shows that when the private signal is perfect, there is no equilibrium, leading to the famous result on the impossibility of informationally efficient markets. In part (b) of Theorem 6, GS further show that when the private signal tends to be perfect, the stock price tends to be fully revealing, but the net trade, which is equal to informed traders' net demand or uninformed traders' net supply, converges to zero. GS then conclude that

> *"Thus, the result that competitive equilibrium is incompatible with informationally efficient markets should be interpreted as meaning that speculative markets where prices reveal a lot of information will be very thin because it will be composed of individuals with very similar beliefs."*

GS further conclude that when the private signal is perfect, the market breaks down naturally due to lack of trade.

However, we find that part (b) of Theorem 6 of GS is incorrect, and thus the associated explanations are also incorrect. Our corrected Theorem 6 shows that when the private signal tends to be perfect, there is still significant net trade in the market, which converges to the innovation of the noisy supply, i.e., the noisy supply minus its expectation. Our corrected Theorem 6 implies that when the market converges to strong-form efficiency and thus traders have almost homogeneous beliefs about the stock payoff, there is still significant net trade. That is, a competitive equilibrium is compatible with an informationally efficient market in the limit. In addition, when the private signal is perfect, the market breaks down due to the competitive assumption rather than "thinness" or "lack of trade." In particular, our results hold for any finite information costs, which affect only the rate of convergence in our various limiting results.

When the price tends to be fully revealing, all traders have almost homogeneous beliefs about the stock payoff. Because traders have the same endowment and the same risk preference, one may think that traders' demands for the stock should converge and thus the net trade in the market should vanish. This thinking, however, is not necessarily correct.

A trader's optimal demand for the stock depends on the trade-off between his conditional expected profit per share and his conditional risk per share. Because the price tends to be fully revealing, the conditional expected profits per share of the informed trader and the uninformed trader converge to zero. Because informed traders observe the private signal directly, the expected profit per share of the informed trader converges to zero in a lower order than that of the uninformed trader. In addition, the conditional risks per share of both the informed and the uninformed trader converge to zero in the same order. As a result, in equilibrium, the informed trader's optimal demand converges to infinity, but the uninformed trader's optimal demand converges to a finite quantity. Consequently, although the fraction of informed traders goes to zero, significant net trade exists in the market.

It is interesting to note that when the price tends to fully reveal the private signal, informed traders are still willing to pay for it. The intuition of this result is as follows. The informed trader observes the private signal directly but the uninformed trader has to learn the private signal from the stock price. When the price tends to be fully revealing, the uninformed trader's information, which is inferred from the price, converges to the true private signal, but it is still infinitesimally inferior to the private signal itself. In other words, the difference between the uninformed trader's information and the private signal converges to zero, but it is not zero. As a result, it remains worthwhile for a small number of traders to buy information because they are able to cover the cost by trading infinitely aggressively to exploit the infinitesimal mispricing that remains.

To extend the corrected Theorem 6, we further show that when the precision of the private signal increases, the stock price tends to reveal more private information, and thus traders hold closer beliefs about the stock payoff, but the net trade may increase rather than decrease.

In addition, we study the limiting case in which traders tend to be risk neutral in the GS model. In this case, the stock price tends to fully reveal the private information, but the net trade in the market is still significant, as in the case where the private signal tends to be perfect. We further show that when traders' risk aversion decreases, the stock price tends to reveal more private information, but the net trade may increase rather than decrease, similar to the case in which the precision of the private signal increases.

Our concept of convergence is similar to those of Chau and Vayanos (2008), Kovalenkov and Vives (2014), and Guo and Ou-Yang (2015). In a strategic trading model, Kovalenkov and Vives show that when traders are risk neutral and the noise in the market goes to infinity, the price tends to be fully revealing. Chau and Vayanos develop a model in which the market tends to be strong-form efficient in the continuous-time limit when the cost of the private information is zero. We believe that our case, in which traders tend to be risk neutral, serves as a competitive counterpart to the Kovalenkov-Vives and Chau-Vayanos models. In addition, Milgrom (1981), Jackson (1992), and Muendler (2007) show that fully revealing prices can be achieved in other setups.

It should be emphasized that while an overall equilibrium is compatible with an informationally efficient market as a limiting result, there is in fact a discontinuity when the private signal is perfect, so our results do not contradict Theorem 5 in the GS paper. Our results imply that one cannot take the limit of the private signal tending to be perfect to understand what happens in the case in which the private signal is perfect. This highlights the fact that one must be careful in interpreting the results of limit economies in rational expectations models with endogenous information acquisition.

2 The Grossman-Stiglitz Model

In this section, we review the setup and some of the key results of the original GS (1980) model.

There is a continuum of traders in a competitive market, who are indexed by $i \in [0, 1]$. Traders have CARA utility: $V(W_i) = -e^{-aW_i}$, where $a > 0$ is the coefficient of absolute risk aversion and W_i is the (net) wealth of the ith trader at the end of the period. There is a safe asset with the return of unity and a risky asset (stock) whose payoff per share, u, is given by $u = \theta + \epsilon$, where $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$ is observable at a constant cost c and $\epsilon \sim N(0, \sigma_\epsilon^2)$ is unobservable. There are two types of traders: informed traders who observe θ and the stock price, and uninformed traders who observe only the stock price. We use subscripts I and U to denote the informed trader and the uninformed trader, respectively. The information sets of traders are denoted as $\mathcal{F}$. Then, $\mathcal{F}_I = \{\theta, P\}$ and $\mathcal{F}_U = \{P\}$. All traders are, *ex ante*, identical. Whether they are informed depends on whether they have spent c to obtain the private signal θ. The fraction of informed traders is denoted as λ. In addition, the per capita noisy supply of the stock is denoted as $x \sim N(\bar{x}, \sigma_x^2)$. θ, ϵ, and x are mutually independent.

According to Theorem 1 of GS, the rational expectations price function is assumed to be

$$P = P_\lambda(\theta, x) = \beta_1\bar{\theta} + \beta_2\theta - \beta_3(x - \bar{x}) - \beta_4\bar{x}, \tag{1}$$

where the $\beta's$ are constants. Maximizing their expected utilities yields the optimal demand for the stock by each informed trader, X_I, and the optimal demand by each uninformed trader, X_U:

$$X_I = \frac{\mathbb{E}(u|\theta, P) - P}{a\mathrm{Var}(u|\theta, P)}, \quad X_U = \frac{\mathbb{E}(u|P) - P}{a\mathrm{Var}(u|P)}. \tag{2}$$

In equilibrium, supply is equal to demand, yielding

$$\lambda X_I + (1-\lambda)X_U = x, \tag{3}$$

and the $\beta's$ can then be determined. The expressions for the $\beta's$ are presented in the online appendix. If $\lambda = 0$, then $\beta_2 = 0$. If $\lambda > 0$, then $t \equiv \beta_3/\beta_2 = a\sigma_\epsilon^2/\lambda$. Therefore, observing P is equivalent to observing $(\theta - tx)$ for $\lambda > 0$.

To simplify the presentation, GS define the following notation:

$$\phi = \left(\frac{a\sigma_\epsilon^2}{\lambda}\right)^2 \frac{\sigma_x^2}{\sigma_\theta^2}, \quad \rho = \frac{\sigma_\theta^2}{\sigma_\epsilon^2}, \tag{4}$$

where ρ represents the precision of the signal. When $\sigma_\epsilon^2 \to 0$, $\rho \to +\infty$; that is, the private signal tends to be perfect. According to Theorem 2 of GS, the ratio of the expected utility of the informed trader, $EV(W_I)$, to that of the uninformed trader, $EV(W_U)$, denoted as $\gamma(\lambda)$, is given as follows:

$$\gamma(\lambda) \equiv \frac{EV(W_I)}{EV(W_U)} = e^{ac}\sqrt{\frac{\mathrm{Var}(u|\theta)}{\mathrm{Var}(u|P)}} = e^{ac}\left(\frac{1+\phi}{1+\phi+\phi\rho}\right)^{1/2}. \tag{5}$$

The overall equilibrium is defined as a pair $(\lambda, P_\lambda(\theta, x))$ such that $0 \le \lambda \le 1$ if $\gamma(\lambda) = 1$ at $P_\lambda(\theta, x)$; $\lambda = 0$ if $\gamma(0) > 1$ at $P_0(\theta, x)$; $\lambda = 1$ if $\gamma(1) < 1$ at $P_1(\theta, x)$.

According to Corollary 1 of GS, $\gamma(\lambda)$ is a strictly increasing function of λ, *ceteris paribus*. Therefore, the unique solution to $\gamma(\lambda^*) = 1$ can be obtained:

$$\lambda^* = \frac{a\sigma_\epsilon\sigma_x}{\sigma_\theta}\left(\frac{\sigma_\theta^2}{e^{2ac}-1} - \sigma_\epsilon^2\right)^{1/2}. \tag{6}$$

If $0 < \lambda^* < 1$, then the condition $\gamma(\lambda^*) = 1$ ensures that the expected utilities of the informed trader and the uninformed trader are equal and the market is in an overall equilibrium. Our paper considers two limiting cases of $\sigma_\epsilon^2 \to 0$ and $a \to 0$, in which the inequality $0 < \lambda^* < 1$ always holds when parameters (σ_θ, σ_x, c, a, σ_ϵ) are strictly positive. Hereafter, unless otherwise specified, all of our analyses are performed in the overall equilibrium, i.e., $\lambda = \lambda^*$.

The price informativeness, Q, is defined as

$$Q = [\mathrm{Corr}(\theta, P)]^2 = \frac{1}{1+\phi}. \tag{7}$$

When $Q = 1$, the price fully reveals the private signal.[1] Substituting the expression for λ given in equation (6) into the expression for Q, we obtain

$$Q = 1 - \frac{(e^{2ac}-1)\sigma_\epsilon^2}{\sigma_\theta^2}. \tag{8}$$

It is clear that when the private signal converges to be perfect ($\sigma_\epsilon^2 \to 0$) or traders tend to be risk neutral ($a \to 0$), the price tends to be fully revealing in the overall equilibrium for any finite information costs.

As we focus mostly on the limiting cases where $\sigma_\epsilon^2 \to 0$ and $a \to 0$, for ease of exposition, we define the notation for the rates of convergence as follows. Suppose that f and g are functions of variable z, and that when z goes to zero, f and g converge to zero or infinity. $f \sim g$ means that $\lim_{z\to 0} f/g = 1$, and $f \propto g$ means that

[1]When the price is fully revealing, it is a sufficient statistic for the private signal θ, which implies that $\mathrm{Var}(\theta|P) = 0$. Note that $\mathrm{Var}(\theta|P) = \mathrm{Var}(\theta|\theta - tx) = \sigma_\theta^2 - \sigma_\theta^4/(\sigma_\theta^2 + t^2\sigma_x^2) = \sigma_\theta^2(1-Q)$. Therefore, $\mathrm{Var}(\theta|P) = 0$ is equivalent to $Q = 1$.

there exists a $K > 0$ such that $\lim_{z \to 0} |f/g| = K$; that is, $|f|$ converges at the same rate as $|g|$.

From equation (6), in the overall equilibrium, $\sigma_\epsilon^2 \to 0$ and $a \to 0$ lead to $\lambda \propto \sigma_\epsilon$ and $\lambda \propto a^{1/2}$, respectively. Based on the rate of $\lambda \to 0$, we further obtain the rates of convergence of t, ϕ, and ρ. The results are summarized in Table 1.

	λ	t	ϕ	ρ
$\sigma_\epsilon^2 \to 0$	σ_ϵ	σ_ϵ	σ_ϵ^2	$1/\sigma_\epsilon^2$
$a \to 0$	$a^{1/2}$	$a^{1/2}$	a	a^0

Table 1: Rates of Convergence of λ, t, ϕ, and ρ

The proofs of Table 1 and all propositions in the paper are presented in the online appendix.

3 Net trade

3.1 Corrected Theorem 6 of Grossman and Stiglitz (1980)

In Section III of GS, they discuss trading activities in the market in the overall equilibrium. GS interpret the noisy supply per capita x as the initial endowment of each trader. They define $(X_I - x)$ as the net trade per capita, which can be understood as the informed trader's net demand, and $\lambda(X_I - x)$ as the net trade in the market. Note that the market clearing condition, $\lambda X_I + (1-\lambda)X_U = x$, is equivalent to $\lambda(X_I - x) = -(1-\lambda)(X_U - x)$. Therefore, the net demand of informed traders, $\lambda(X_I - x)$, is equal to the net supply of uninformed traders, $-(1-\lambda)(X_U - x)$, and the net trade is the trade between informed and uninformed traders.

GS calculate the net trade per capita and the variance of the net trade in equations (GS 22) and (GS 24) as follows:

$$X_I - x = \frac{1-\lambda}{1+\phi+\lambda\phi\rho}\left[\left(\phi\rho + \frac{a\sigma_\epsilon^2}{\lambda}\right)(x-\bar{x}) + [(\phi+1)\rho - 1](\theta - \bar{\theta}) + \phi\rho\bar{x}\right], \tag{GS 22}$$

$$\mathrm{Var}[\lambda(X_I - x)] = \frac{\sigma_\theta^2(1-\lambda)^2\lambda^2}{(1+\phi+\lambda\phi\rho)^2\rho^2}\left[[(\phi+1)\rho-1]^2 + \left(\phi\rho + \frac{a\sigma_\epsilon^2}{\lambda}\right)^2 \frac{\sigma_x^2}{\sigma_\theta^2}\right]. \quad \text{(GS 24)}$$

Based on the above two equations, GS conclude that when σ_ϵ^2 goes to zero, the mean and variance of $\lambda(X_I - x)$ converge to zero, leading to Part (b) of their Theorem 6.[2]

> ***Theorem 6 of GS***: *(a) $\cdots$. (b) As the precision of informed traders' information ρ goes to infinity, the mean and variance of trade go to zero.*

GS argue that trade stems from the differences in endowments, preferences, or beliefs among traders. Therefore, their explanation for Theorem 6 is that because traders have almost homogenous beliefs about the stock payoff, the net trade in the market tends to vanish.

However, we find that equations (GS 22) and (GS 24) are incorrect. The correct versions of them are given as follows:

$$\begin{aligned} X_I - x &= (1-\lambda)\left[\frac{(\phi\rho + 1/\lambda)(x-\bar{x}) + \phi(\theta-\bar{\theta})/(a\sigma_\epsilon^2) + \phi\rho\bar{x}}{1+\phi+\lambda\phi\rho}\right], && (9) \\ \mathrm{Var}[\lambda(X_I - x)] &= \lambda^2(1-\lambda)^2\left[\frac{(\phi\rho+1/\lambda)^2\sigma_x^2 + [\phi/(a\sigma_\epsilon^2)]^2\sigma_\theta^2}{(1+\phi+\lambda\phi\rho)^2}\right]. && (10) \end{aligned}$$

Taking the limit as $\sigma_\epsilon^2 \to 0$ in equation (9) and using the results in Table 1, we then obtain the correct version of Theorem 6 in GS (1980) as follows.

Corrected Theorem 6 of GS (1980): *In the overall equilibrium, when σ_ϵ^2 goes to zero, the net trade converges to the innovation of the noisy supply state by state, i.e.,*

$$\lim_{\sigma_\epsilon^2 \to 0} \lambda(X_I - x) = x - \bar{x}. \quad (11)$$

Thus, when σ_ϵ^2 goes to zero, the mean and variance of the net trade go to zero and σ_x^2, respectively.

[2]In the original GS paper, they let $h = \sigma_\epsilon^2$. For ease of notation, we do not introduce h in our paper.

The corrected Theorem 6 suggests that when the private signal tends to be perfect, there is still significant net trade in the market. From equation (6), when $\sigma_\epsilon^2 \to 0$, λ converges to zero. Therefore, the corrected Theorem 6 implies that when $\sigma_\epsilon^2 \to 0$, $\lambda X_I \to x - \bar{x}$ and $(1-\lambda)X_U \to \bar{x}$. That is, informed traders hold the entire innovation of the noise trading, $x - \bar{x}$, but uninformed traders hold just the expectation of the noise, $\bar{x}$. It is interesting that the fraction of informed traders converges to zero, but the overall quantity traded by all informed traders still limits to a finite value.

Theorem 5 of GS shows that an overall equilibrium does not exist or the market breaks down at $\sigma_\epsilon^2 = 0$. Based on the incorrect result in Theorem 6 of their paper, GS conclude that a competitive equilibrium is incompatible with informationally efficient markets, because when the market converges to strong-form efficiency, it becomes very thin. In addition, GS explain that the market moves from "thinness" or "lack of trade" (when $\sigma_\epsilon^2 \to 0$) to breakdown (at $\sigma_\epsilon^2 = 0$) naturally.

The corrected Theorem 6 demonstrates that the market is not thin when it converges to strong-form efficiency. That is, a competitive equilibrium is compatible with an informationally efficient market in the limit. In addition, because the market breaks down at $\sigma_\epsilon^2 = 0$, we cannot discuss the net trade in this case. Therefore, it is inappropriate to conclude that the market breaks down due to "thinness" or "lack of trade." In particular, these results hold for any finite information costs.

3.2 Why is there significant trade between traders?

Rearranging the market clearing condition, $\lambda X_I + (1-\lambda)X_U = x$, we obtain

$$\lambda(X_I - x) = \lambda(1-\lambda)(X_I - X_U). \tag{12}$$

Therefore, the net trade is partially determined by how much X_I and X_U differ from each other. When traders have identical beliefs about the stock payoff, we have

$X_I = X_U$, and thus there is no trade between informed and uninformed traders, but when traders have different beliefs, there is trade. It seems logical that when traders' beliefs converge, the net trade should tend to vanish. Our paper, however, demonstrates that this thinking is not necessarily correct.

Recall that

$$X_I = \frac{\mathbb{E}(u|\mathcal{F}_I) - P}{a\mathrm{Var}(u|\mathcal{F}_I)}, \quad X_U = \frac{\mathbb{E}(u|\mathcal{F}_U) - P}{a\mathrm{Var}(u|\mathcal{F}_U)}.$$

Clearly, the optimal demand of trader i depends on the trade-off between his conditional expected profit per share, $\mathbb{E}(u|\mathcal{F}_i) - P$, and his conditional risk per share, $\mathrm{Var}(u|\mathcal{F}_i)$. For ease of exposition, we define the expected profit per share of trader i conditional on his information set as $CEPS_i = \mathbb{E}[(u - P)|\mathcal{F}_i]$.

Proposition 1 *In the overall equilibrium, when $\sigma_\epsilon^2 \to 0$, we have the following results.*

$$\lambda \sim \frac{a\sigma_\epsilon \sigma_x}{(e^{2ac} - 1)^{1/2}} \propto \sigma_\epsilon, \tag{13}$$

$$CEPS_I = \mathbb{E}(u|\theta) - P \sim \frac{a\sigma_\epsilon^2}{\lambda}(x - \bar{x}) \propto \sigma_\epsilon, \tag{14}$$

$$CEPS_U = \mathbb{E}(u|P) - P \sim a\sigma_\epsilon^2 e^{2ac}\bar{x} \propto \sigma_\epsilon^2, \tag{15}$$

$$\mathrm{Var}(u|\theta) = \sigma_\epsilon^2, \quad \mathrm{Var}(u|P) = e^{2ac}\sigma_\epsilon^2 \propto \sigma_\epsilon^2. \tag{16}$$

Consequently, we obtain that

$$X_I \sim \frac{x - \bar{x}}{\lambda} \propto 1/\sigma_\epsilon, \quad X_U \sim \bar{x}. \tag{17}$$

According to equation (8), when σ_ϵ^2 goes to zero, the stock price tends to be fully revealing; that is, $P - \theta \to 0$. Therefore, $\mathbb{E}(u|\theta) - \mathbb{E}(u|P) \to 0$ and $\mathrm{Var}(u|\theta) - \mathrm{Var}(u|P) \to 0$, or all traders have almost homogeneous beliefs about u. Note that $\mathbb{E}(u|\theta) = \theta$, so both $CEPS_I$ and $CEPS_U$ converge to zero. In addition, because the private signal tends to be perfect, the conditional risks, $\mathrm{Var}(u|\theta)$ and $\mathrm{Var}(u|P)$, converge to zero.

Notice that informed traders observe the private signal directly, but uninformed traders observe only the stock price, which is equivalent to the private signal minus a noise term ($\{P\} \equiv \{\theta - tx\}$). When the price tends to be fully revealing, tx goes toward zero but is not zero, so the informed trader still holds an infinitesimal informational advantage over the uninformed trader, even though this advantage vanishes in the limit. Therefore, $CEPS_I$ converges to zero as σ_ϵ^2 goes to zero at a rate smaller than $CEPS_U$. Conversely, $\mathrm{Var}(u|\theta)$ and $\mathrm{Var}(u|P)$ converge to zero in the same order as σ_ϵ^2. Consequently, even when traders tend to have homogeneous beliefs about u, $X_I \propto 1/\sigma_\epsilon$ but $X_U \to \bar{x}$, leading to significant net trade.

It should be noted that in this limiting case, $\mathrm{Var}(u|\theta) \to 0$ and $\mathrm{Var}(u|P) \to 0$ are necessary conditions for the significant net trade to arise. If traders' beliefs converge but $\mathrm{Var}(u|\theta)$ and $\mathrm{Var}(u|P)$ were to converge to finite values rather than zero, then both X_I and X_U would converge to each other and thus the net trade would converge to zero.

Another interesting observation is that when the private signal tends to be perfect, the aggregate demand of informed traders is independent of the private signal. That is, informed traders tend to stop speculating on their private information once it is revealed, and instead, they play the role of "market making."[3]

Furthermore, it should be emphasized that the trade between informed and uninformed traders is due to information asymmetry rather than risk sharing. Because all traders have the same endowments and risk aversion, if the trade were driven by risk sharing, then the traders' demand for the stock would be identical, but this is

[3]To some extent, this result is the opposite of Example 4.3 of Vives (2008), in which informed traders withhold from market making and speculate only on his private information.

not the case here.

3.3 Further discussion on the net trade

To extend the result of the corrected Theorem 6, in this section we discuss how the net trade changes when traders' beliefs about the payoff approach each other gradually, but remain away from the limiting case.

Proposition 2 *In the overall equilibrium, if $\sigma_\epsilon^2 < \sigma_\theta^2/[2(e^{2ac}-1)]$, then the variance of the net trade in the market increases with the precision of the private signal.*

Note that when the precision of the private signal increases, the price becomes more informative. Proposition 2 then suggests the counterintuitive result that when traders hold closer beliefs about the payoff, the net trade in the market may increase rather than decrease.

According to equation (12), the variance of the net trade is given by

$$\mathrm{Var}[\lambda(X_I - x)] = \lambda^2(1-\lambda)^2\mathrm{Var}(X_I - X_U). \tag{18}$$

Based on the above equation, the net trade depends on two components: $\lambda^2(1-\lambda)^2$ and $\mathrm{Var}(X_I - X_U)$. The first component depends on the population of informed and uninformed traders, and the second represents the difference between the demand of each informed trader and that of each uninformed trader.

For a sufficiently small σ_ϵ^2, as σ_ϵ^2 goes down, the fraction of informed traders decreases because the price is increasingly informative (the remaining informed traders face a lower conditional risk per share, relative to the conditional expected profit per share, and therefore trade more aggressively), so fewer traders find it optimal to buy a private signal. In this case, $\lambda^2(1-\lambda)^2$ increases with λ, and thus increases with σ_ϵ^2. However, due to the aggressive trading behavior of the remaining informed traders,

X_I and X_U diverge from each other. It turns out that for a sufficiently small σ_ϵ^2, the second effect dominates, so the net trade actually increases as the private signal becomes more precise ($\sigma_\epsilon^2 \downarrow$).[4] Therefore, when the precision of the signal increases, the stock price becomes more informative, and thus traders hold closer beliefs about the payoff, but the net trade increases.

In fact, the empirical findings of Brockman and Yan (2009) and Gul et al. (2010) provide some supportive evidence for the result of Proposition 2. Stock return nonsynchronicity is widely used as a measure of the price informativeness, as first proposed by Roll (1988) and further developed by Morck et al. (2000), Durnev et al. (2003), and Durnev et al. (2004). According to these studies, the variation of a stock return can be decomposed into three components: market-related variation, industry-related variation, and firm-specific variation. The first two components measure systematic variations, while the third captures firm-specific variation or price nonsynchronicity.[5] Using stock return nonsynchronicity as a proxy for price informativeness, Brockman and Yan (2009) and Gul et al. (2010) find that trading volume turnover is positively correlated with stock price informativeness. Because the trading volumes of different sized firms are not comparable, we interpret the net trade as the trading volume turnover. Therefore, our result of Proposition 2 that the net trade may increase with

[4] When traders' beliefs converge, if the limits of $\mathrm{Var}(u|\theta)$ and $\mathrm{Var}(u|P)$ are nonzero, then X_I eventually approaches X_U and the net trade eventually decreases with the stock price informativeness. For instance, we find that when the cost of the private signal decreases, the price informativeness increases, and $\mathrm{Var}(u|\theta)$ and $\mathrm{Var}(u|P)$ do not converge to zero in the limit of the cost going to zero. In this case, traders hold closer beliefs about the payoff, and the net trade decreases for some parameter values. In contrast, if the limits of $\mathrm{Var}(u|\theta)$ and $\mathrm{Var}(u|P)$ are zero, then when $\mathrm{Var}(u|\theta)$ and $\mathrm{Var}(u|P)$ decrease, informed traders may trade more aggressively than uninformed traders, so that X_I and X_U are likely to diverge from each other and the net trade may increase, as in the case where the precision of the private signal increases.

[5] Empirical studies find some supporting evidence for the validity of using stock return nonsynchronicity as a proxy for the price informativeness. For instance, Durnev et al. (2003) find that stock price nonsynchronicity is highly correlated with stock prices' ability to predict firms' future earnings, supporting the argument that price nonsynchronicity reflects private information more than noise.

the stock price informativeness is consistent with the empirical findings of Brockman and Yan (2009) and Gul et al. (2010).

3.4 Why do traders pay for private information?

We have obtained explicit solutions to the equilibrium when σ_ϵ^2 goes to zero. By construction, the expected utility of the informed trader is equal to that of the uninformed trader in the overall equilibrium. This is why the informed trader is still willing to pay the cost c for private information. When the stock price tends to be fully revealing, however, one may still wonder how the informed trader can cover the information cost in equilibrium.

Note that trader i's expected utility is given by

$$\mathbb{E}\left[-\exp(-aW_i)\right] = \mathbb{E}\left\{-\exp\left[\mathbb{E}(-aW_i|\mathcal{F}_i) + \frac{1}{2}\mathrm{Var}(-aW_i|\mathcal{F}_i)\right]\right\}. \tag{19}$$

The uninformed trader's net profit is given by $W_U = X_U(u-P)$. When σ_ϵ^2 goes to zero, $X_U \sim \bar{x}$, $\mathbb{E}(u|P) - P \propto \sigma_\epsilon^2$, and $\mathrm{Var}(u|P) \propto \sigma_\epsilon^2$, so we have

$$\lim_{\sigma_\epsilon^2 \to 0} \mathbb{E}(-aW_U|P) = 0 \text{ and } \lim_{\sigma_\epsilon^2 \to 0} \mathrm{Var}(-aW_U|P) = 0.$$

Therefore, when σ_ϵ^2 goes to zero, $\mathbb{E}[-\exp(-aW_U)] = -1$.

The informed trader's net profit is given by $W_I = X_I(u-P) - c$. From equations (14) and (17), we can obtain

$$\begin{aligned}\lim_{\sigma_\epsilon^2 \to 0} \mathbb{E}(-aW_I|\mathcal{F}_I) &= -\frac{e^{2ac}-1}{\sigma_x^2}(x-\bar{x})^2 + ac,\\ \lim_{\sigma_\epsilon^2 \to 0} \mathrm{Var}(-aW_I|\mathcal{F}_I) &= \frac{e^{2ac}-1}{\sigma_x^2}(x-\bar{x})^2.\end{aligned}$$

Substituting λ in equation (13) into $\mathbb{E}(-aW_I|\mathcal{F}_I)$ and $\mathrm{Var}(-aW_I|\mathcal{F}_I)$, we have

$$\lim_{\sigma_\epsilon^2 \to 0} \mathbb{E}\left\{-\exp\left[\mathbb{E}(-aW_I|\mathcal{F}_I) + \frac{1}{2}\mathrm{Var}(-aW_I|\mathcal{F}_I)\right]\right\} = -1.$$

To sum up, when σ_ϵ^2 goes to zero, $(\theta - P) \to 0$, but X_I goes to infinity. Hence, both the conditional expectation and the conditional variance of the informed trader's net profit are nonzero in the limit, but they offset each other exactly when taking the unconditional expectation. Consequently, the expected utilities of informed and uninformed traders are equal.

As we explained in the introduction, when the price tends to be fully revealing, the uninformed trader's information, which is inferred from the price, converges to the true private signal, but it is still infinitesimally inferior to the private signal itself. As a result, it remains worthwhile for a small number of traders to buy information because they are able to cover the cost by trading infinitely aggressively to exploit the infinitesimal mispricing that remains.

The intuition of our case is similar to those in Chau and Vayanos (2008) and Kovalenkov and Vives (2014). In their strategic trading models, when the price tends to be fully revealing, the conditional expected profit per share of the risk-neutral informed trader goes to zero, but his demand goes to infinity, so that his expected gross profit is still positive.

4 Another limiting case: traders tending to be risk neutral

Risk-neutral traders are widely assumed in strategic trading models, but they are avoided in competitive trading models due to the breakdown of equilibria. In this section, we consider the limiting case in which traders tend to be risk neutral. We summarize the relevant results in the following proposition.

Proposition 3 *In the overall equilibrium, when $a \to 0$, we have the following results.*

$$\lambda \propto a^{1/2}. \tag{20}$$

$$CEPS_I = \mathbb{E}(u|\theta) - P \propto a^{1/2}, \quad CEPS_U = \mathbb{E}(u|P) - P \propto a. \tag{21}$$

$$X_I \propto 1/a^{1/2}, \quad X_U \sim \bar{x}. \tag{22}$$

$$\lim_{a\to 0} \ \lambda(X_I - x) = x - \bar{x}. \tag{23}$$

Recall that when traders tend to be risk neutral, the stock price tends to be fully revealing. Because the informed trader observes the private signal directly but the uninformed trader observes only the price, $CEPS_I$ and $CEPS_U$ converge to zero in the orders of $a^{1/2}$ and a, respectively. Conversely, the risk aversion of the traders goes to zero in the same order of a. As a result, $X_I \propto 1/a^{1/2}$, $X_U \sim \bar{x}$, and the net trade is significant. Proposition 3 then shows that when $a \to 0$, the market converges to strong-form efficiency, and thus traders hold almost homogenous beliefs, but there is also significant net trade in the market, which converges to the innovation of the noisy supply, $(x - \bar{x})$.

Finally, we consider the relation between the net trade and the risk aversion.

Proposition 4 *When there is a low level of risk aversion, the variance of the net trade in the market decreases with the risk aversion of traders.*

Proposition 4 illustrates that when traders' risk aversion decreases, the stock price is more informative, and thus traders hold closer beliefs about the stock payoff, but the net trade may increase.

5 Conclusion

In this paper, we find that part (b) of Theorem 6 of GS is incorrect. Our corrected Theorem 6 demonstrates that when the private signal tends to be perfect, the market converges to strong-form efficiency, and thus traders have almost homogeneous beliefs about the stock payoff, but there is still significant trade between informed and uninformed traders for any finite information costs. That is, a competitive equilibrium is compatible with an informationally efficient market in the limit. Similarly, we find that when traders tend to be risk neutral, the market converges to strong-form efficiency, and there is also significant net trade in the market. We further show that when the stock price becomes more informative, and thus traders' beliefs about the stock payoff become closer, the market may not become thinner, and the net trade may even increase.

6 References

Brockman, P., and X. Yan, 2009, Block ownership and firm-specific information, *Journal of Banking and Finance*, 33, 308-316.

Chau, M., and D. Vayanos, 2008, Strong-form efficiency with monopolistic insiders, *Review of Financial Studies*, 21, 2275-2306.

Durnev, A., R. Morck, and B. Yeung, 2004, Value enhancing capital budgeting and firm-specific stock return variation, *Journal of Finance*, 59, 65-105.

Durnev, A., R. Morck, B. Yeung, and P. Zarowin, 2003, Does greater firm-specific return variation mean more or less informed stock pricing, *Journal of Accounting Research*, 41, 797-836.

Grossman, S. J., and J. E. Stiglitz, 1980, On the impossibility of informationally efficient markets, *American Economic Review*, 70, 393-408.

Gul, F. A., J. Kim, and A. A. Qiu, 2010, Ownership concentration, foreign shareholding, audit quality, and stock price synchronicity: Evidence from China, *Journal of Financial Economics*, 95, 425-442.

Guo, M., and H. Ou-Yang, 2015, Feedback trading between fundamental information and non-fundamental information, *Review of Financial Studies*, 28, 247-296.

Jackson, M. O., 1991, Equilibrium, price formation, and the value of private information, *Review of Financial Studies*, 4, 1-16.

Kovalenkov, A., and X. Vives, 2014, Competitive rational expectations equilibria without apology, *Journal of Economic Theory*, 149, 211-235.

Milgrom, P. R., 1981, Rational expectations, information acquisition, and competitive bidding, *Econometrica*, 49, 921-943.

Morck, R., B. Yeung, and W. Yu, 2000, The information content of stock markets: Why do emerging markets have synchronous stock price movements? *Journal of Financial Economics*, 59, 215-260.

Muendler, M. A., 2007, The possibility of informationally efficient markets, *Journal of Economic Theory*, 133, 467-483.

Roll, R., 1988, R^2, *Journal of Finance*, 43, 541-566.

Vives, X., 2008, Information and learning in markets: The impact of market microstructure, *Princeton University Press.*

7 Online Appendix

7.1 Basic Mathematical Background

We first present some basic mathematical relations necessary for the proofs of the propositions in this paper.

Based on equation (6), when $\sigma_\epsilon^2 \to 0$, we obtain

$$\lambda^* = \frac{a\sigma_\epsilon \sigma_x}{\sigma_\theta}\left(\frac{\sigma_\theta^2}{e^{2ac}-1} - \sigma_\epsilon^2\right)^{1/2} \sim \frac{a\sigma_\epsilon\sigma_x}{(e^{2ac}-1)^{1/2}} \to 0 \text{ and } \lambda^* \propto \sigma_\epsilon. \tag{24}$$

When $a \to 0$, we first have $e^{2ac} \sim (1+2ac)$, and we then obtain

$$\lambda^* = \frac{a\sigma_\epsilon \sigma_x}{\sigma_\theta}\left(\frac{\sigma_\theta^2}{e^{2ac}-1} - \sigma_\epsilon^2\right)^{1/2} \sim \frac{a^{1/2}\sigma_\epsilon\sigma_x}{(2c)^{1/2}} \to 0 \text{ and } \lambda^* \propto a^{1/2}. \tag{25}$$

Recall that

$$t = \frac{a\sigma_\epsilon^2}{\lambda}, \quad \phi = \left(\frac{a\sigma_\epsilon^2}{\lambda}\right)^2 \frac{\sigma_x^2}{\sigma_\theta^2}, \quad \rho = \frac{\sigma_\theta^2}{\sigma_\epsilon^2}.$$

When $\sigma_\epsilon^2 \to 0$ and $a \to 0$, based on the rate of $\lambda \to 0$, we obtain the rates of convergence of t, ϕ, and ρ. The results in Table 1 can be obtained.

Following GS, we assume the price function as

$$P = P_\lambda(\theta, x) = \beta_1\bar{\theta} + \beta_2\theta - \beta_3(x-\bar{x}) - \beta_4\bar{x}.$$

Imposing the market-clearing condition, standard computations deliver the coefficients in the price function as follows:

$$\begin{aligned}
\beta_1 &= \phi(1-\lambda)/Z, & (26)\\
\beta_2 &= (1+\lambda\phi+\lambda\phi\rho)/Z, & (27)\\
\beta_3 &= t(1+\lambda\phi+\lambda\phi\rho)/Z, & (28)\\
\beta_4 &= a\sigma_\epsilon^2(1+\phi+\phi\rho)/Z, & (29)\\
Z &= 1+\phi+\lambda\phi\rho. & (30)
\end{aligned}$$

Note that $\beta_1 + \beta_2 = 1$ and $\beta_3 = t\beta_2$. In the overall equilibrium, when $\gamma(\lambda) = 1$, we obtain $(1+\phi+\phi\rho) = e^{2ac}(1+\phi)$ by equation (5). Hence, $\beta_4 = a\sigma_\epsilon^2 e^{2ac}(1+\phi)/Z$ in the overall equilibrium.

When σ_ϵ^2 or a goes to zero, λ, ϕ, and $\lambda\phi\rho$ go to zero, leading to

$$1/Z = (1+\phi+\lambda\phi\rho)^{-1} \sim 1-\phi-\lambda\phi\rho.$$

We then have

$$\begin{aligned}
\beta_1 &= \phi(1-\lambda)/Z \sim \phi(1-\lambda)(1-\phi-\lambda\phi\rho) \sim \phi, \\
\beta_2 &= 1-\beta_1 \sim 1-\phi, \\
\beta_3 &= t\beta_2 \sim t(1-\phi), \\
\beta_4 &= a\sigma_\epsilon^2 e^{2ac}(1+\phi)/Z \sim a\sigma_\epsilon^2 e^{2ac}(1+\phi)(1-\phi-\lambda\phi\rho) \sim a\sigma_\epsilon^2 e^{2ac}.
\end{aligned}$$

In addition, recall that $Q = 1/(1+\phi)$. We have

$$\begin{aligned}
Q-\beta_2 &= -\frac{\phi\lambda(1+\phi+\phi\rho)}{(1+\phi)Z} = -e^{2ac}\lambda\phi/Z, \\
Qt-\beta_3 &= t(Q-\beta_2) = -e^{2ac}\lambda\phi t/Z.
\end{aligned}$$

When $\sigma_\epsilon^2 \to 0$ or $a \to 0$, we have

$$\begin{aligned}
Q-\beta_2 &= -e^{2ac}\lambda\phi/Z \sim -e^{2ac}\lambda\phi(1-\phi-\lambda\phi\rho) \sim -e^{2ac}\lambda\phi, \\
Qt-\beta_3 &\sim -e^{2ac}\lambda\phi t.
\end{aligned}$$

We summarize the results in Table 2.

	β_1	β_2	β_3	β_4	$Q-\beta_2$	$Qt-\beta_3$
$\sigma_\epsilon^2 \to 0$	ϕ	$1-\phi$	$t(1-\phi)$	$a\sigma_\epsilon^2 e^{2ac}$	$-e^{2ac}\lambda\phi$	$-e^{2ac}\lambda\phi t$
	σ_ϵ^2	1	σ_ϵ	σ_ϵ^2	$-\sigma_\epsilon^3$	$-\sigma_\epsilon^4$
$a \to 0$	ϕ	$1-\phi$	$t(1-\phi)$	$a\sigma_\epsilon^2 e^{2ac}$	$-e^{2ac}\lambda\phi$	$-e^{2ac}\lambda\phi t$
	a	1	$a^{1/2}$	a	$-a^{3/2}$	$-a^2$

Table 2: Results for the β's and Related Variables in the Limiting Cases

7.2 Proof of Corrected Theorem 6 of GS (1980)

Note that

$$\mathbb{E}(u|\theta,P) = \mathbb{E}(u|\theta) = \theta, \quad \mathrm{Var}(u|\theta,P) = \mathrm{Var}(u|\theta) = \sigma_\epsilon^2, \tag{31}$$

$$\mathbb{E}(u|P) = \mathbb{E}(u|\theta - tx) = \bar\theta + Q[(\theta-\bar\theta) - t(x-\bar x)], \quad \mathrm{Var}(u|P) = \sigma_\epsilon^2\left(\frac{1+\phi+\phi\rho}{1+\phi}\right). \tag{32}$$

Substituting the expression for X_U given in equation (2) into the market clearing condition, $\lambda X_I + (1-\lambda)X_U = x$, we have

$$\lambda X_I + (1-\lambda)\frac{\bar{\theta} + Q(\theta - \bar{\theta}) - Qt(x - \bar{x}) - P}{a\sigma_\epsilon^2(1+\phi+\phi\rho)/(1+\phi)} = x. \tag{33}$$

From the expression for X_I given in equation (2), we have $P = \theta - a\sigma_\epsilon^2 X_I$. Substituting P into equation (33), we have

$$\lambda X_I + (1-\lambda)\frac{(1+\phi)(\bar{\theta} - \theta + a\sigma_\epsilon^2 X_I) + (\theta - \bar{\theta}) - t(x - \bar{x})}{a\sigma_\epsilon^2(1+\phi+\phi\rho)} = x. \tag{34}$$

Rearranging the above equation yields the correct versions of equations (GS 22) and (GS 24).

7.3 Proof of Proposition 1

From the expressions for $\mathbb{E}(u|\theta)$ and $\mathbb{E}(u|P)$ in equations (31) and (32) and the price function in equation (1), we obtain the $CEPS$s of the informed trader and the uninformed trader:

$$\begin{aligned} CEPS_I &= \mathbb{E}(u|\theta) - P = \beta_1(\theta - \bar{\theta}) + \beta_3(x - \bar{x}) + \beta_4\bar{x}, & (35)\\ CEPS_U &= \mathbb{E}(u|P) - P = (Q - \beta_2)(\theta - \bar{\theta}) - (Qt - \beta_3)(x - \bar{x}) + \beta_4\bar{x}. & (36) \end{aligned}$$

Considering the results in Tables 1 and 2, substituting them into equations (35) and (36), and discarding the higher orders of infinitesimals, we have

$$\begin{aligned} \mathbb{E}(u|\theta) - P &= \beta_1(\theta - \bar{\theta}) + \beta_3(x - \bar{x}) + \beta_4\bar{x} \\ &\sim \phi(\theta - \bar{\theta}) + t(1-\phi)(x - \bar{x}) + a\sigma_\epsilon^2 e^{2ac}\bar{x} \\ &\sim \phi(\theta - \bar{\theta}) + t(x - \bar{x}) + a\sigma_\epsilon^2 e^{2ac}\bar{x} \\ &\sim \frac{a\sigma_\epsilon^2}{\lambda}(x - \bar{x}), \end{aligned}$$

$$\begin{aligned} \mathbb{E}(u|P) - P &= (Q - \beta_2)[(\theta - \bar{\theta}) - t(x - \bar{x})] + \beta_4\bar{x} \\ &\sim -e^{2ac}\lambda\phi[(\theta - \bar{\theta}) - t(x - \bar{x})] + a\sigma_\epsilon^2 e^{2ac}\bar{x} \\ &\sim a\sigma_\epsilon^2 e^{2ac}\bar{x}. \end{aligned}$$

From the overall equilibrium condition in equation (5), we have

$$\gamma(\lambda) = e^{ac}\sqrt{\frac{\mathrm{Var}(u|\theta)}{\mathrm{Var}(u|P)}} = 1 \quad \Rightarrow \quad \mathrm{Var}(u|P) = e^{2ac}\mathrm{Var}(u|\theta) = e^{2ac}\sigma_\epsilon^2.$$

Note that $X_I = [\mathbb{E}(u|\theta) - P]/[a\text{Var}(u|\theta)]$ and $X_U = [\mathbb{E}(u|P) - P]/[a\text{Var}(u|P)]$. We then have

$$\begin{aligned} X_I &\sim \frac{x - \bar{x}}{\lambda}, & (37) \\ X_U &\sim \bar{x}. & (38) \end{aligned}$$

This completes the proof of Proposition 1.

7.4 Proof of Proposition 2

The variance of the net trade given in equation (10) can be simplified as follows:

$$Var[\lambda(X_I - x)] = (1-\lambda)^2 \left[\frac{(1+\lambda\phi\rho)^2 + \phi}{(1+\phi+\lambda\phi\rho)^2} \right] \sigma_x^2. \quad (39)$$

From equation (6), we obtain that when $\sigma_\epsilon^2 < \sigma_\theta^2/[2(e^{2ac}-1)]$, λ increases with σ_ϵ^2. Note that both $\phi = t^2\sigma_x^2/\sigma_\theta^2$ and $\lambda\rho = a\sigma_\theta^2/t$ are functions of t, which increases with σ_ϵ. It can be proven that $[(1+\lambda\phi\rho)^2 + \phi]/(1+\phi+\lambda\phi\rho)^2$ decreases with t, so it also decreases with σ_ϵ^2.

7.5 Proof of Proposition 3

1. Equation (20) was proven in Table 1.

2. By repeating the procedures in the proof of Proposition 1, equations (21) and (22) can be obtained.

3. Taking the limit of $a \to 0$ in equation (9) and using the results in Table 1, we obtain equation (23).

7.6 Proof of Proposition 4

When the risk aversion is small enough, we know that equation (25) holds. Substituting it into equation (39) and taking the derivative with respect to a, we can show that the variance of the net trade decreases with a.